Λογαριθμοτεχνία,

OR
The Conſtruction, and uſe
of the Logarithmeticall
TABLES;

By the help of which, Multiplication is performed by Addition, Diviſion by Subſtraction, the extraction of the Square root by bipartition, and of the Cube root by tripartition, &c. Finally, the Golden rule, and the reſolution of Triangles, as well right lined, as Sphericall by Addition and Subſtraction.

Firſt publiſhed in the French Tongue by
Edmund Wingate, an Engliſh Gentleman:

AND

After tranſlated into Engliſh by the ſame Author.

The ſecond Edition, diligently corrected,
and much enlarged by the author
himſelfe.

Ἐὰν ἧς φιλομαθὴς, ἔσῃ πολυμαθής.

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TO THE MOST
High, and Puiſſant Prince
JEAN BAPTISTE,
Gaſton de France, the Kings
onely brother, Duke of
Anjou, &c.

MOſt Illuſtrious Prince) the good ſucceſſe I had a few moneths agoe, in preſenting unto your *Highneſſe* a Treatiſe, that contained the uſes of a new *Rule of Proportion*, hath made me adventure to preſent you with this alſo, which containes the ground, and *conſtruction* thereof, and beſides, the *demonſtration* of all that is therein handled: True it is, I feared to ſeem raſh to appeare ſo often before your *Highneſſe*, yet (conſidering the quality of the ſubject) I thought *this* no leſſe *yours* then the *former*, they having inſeparable dependancy one upon another: Wherefore I promiſe my ſelfe this hope, that although I cannot now avoide the *cenſure* of raſhneſſe, yet have I this to comfort me, that herein I have aſſaied to doe that, whereunto I was wholy obliged. Your *Highneſſe* may excuſe the meaneneſſe of the preſent, For great *Princes*, ſuch as your *Highneſſe* cannot expect from their inferiours, preſents anſwerable to the *greatneſſe* of their quality; but the *Law* that ſhould not admit preſents of any other ſort, would both deprive them of the *acknowledgement* due unto them, and their Subjects of the *contentment* they may receive to make appeare the ardent devotion they have to render them *ſervice*, which I have enforced my ſelfe to doe with the greateſt affection, and ſubmiſſion, that is poſſible for *him* to performe, who deſires for ever to remaine

Your Highneſſe in
all ſervice and
loyalty devoted,

EDM. WINGATE.

The Preface.

FOR *as much as amongſt many inventions, that concerne the* Mathematicks, *none can be found comparable to this of the* Logarithmes, *the worthy labours of thoſe Learned men which have endevoured to advance it, are to be prized accordingly: Theſe are, firſt* John Neper, *Baron of* Merchiſton *in* Scotland, *who hath due right to challenge the firſt invention of the* Logarithmes *in generall; Then Maſter* Henry Brigges, *Profeſſor of Geometry in the Vniverſitie of* Oxford, *to whom is duely attributed the invention, and* *fabrique of that kind of* Logarithmes, *of which we treat in this inſuing Diſcourſe, and which are farre more expedite, then thoſe of Maſter* Nepers *invention; Laſtly, Maſter* Edmund Gunter, *Profeſſor of Aſtronomie in* Greſham *Colledge in* London, *who hath taken great paines in calculating of a table, contayning the* Logarithmes *of the* Sines, *and* Tangents *of all the degrees and minutes of the Quadrant. Now this Treatiſe following, is nothing elſe but an orderly* Compendium *as well of the conſtruction, as alſo of the joynt and ſeverall uſes of Maſter* Brigges *his Logarithmes, and Maſter* Gunters *ſaid Table: For although the inſuing Tables (now publiſhed in this Edition) be not the very ſame with theirs, yet are they taken and collected out of theirs, and do all participate of the ſelf ſame nature in operation; And concerning the ſame Treatiſe; you* *ſhall further obſerve, that the three firſt Chapters thereof are nothing elſe, but an Abridgement or rather a briefe explanation of that, which Maſter* Brigges *delivers in his Booke (intitled* Arithmetica Logarithmica) *wherein I produce no other Tables or Examples, then thoſe which he there uſeth; not onely becauſe they are apteſt to expreſſe the ſubjected matter, but likewiſe to the end that this Tract might ſerve as an Introduction to that learned Worke of his.*

The occaſion of **compoſing** *this Treatiſe was this: In* Anno 1624. *I making a journey into* France, *had the happineſſe to be the firſt tranſporter of the uſe of theſe inventions into thoſe parts; Where as ſoone as I was arrived, divers Mathematicians of the chiefeſt note in* Paris, *reſorting to my chamber, and I communicating unto them firſt the manifold uſes of the*

Logarithmes *deſcribed upon Maſter* Gunters Croſſe-ſtaffe*, they earneſtly importuned mee to expreſſe them by ſome ſhort Tractate in the* French *Tongue, which when I had compoſed, and prefixed thereunto a Preface declaring the whole Hiſtory of the rare and exquiſite Invention of* Logarithmes, (viz. *ſuch as you finde at the beginning of this) as alſo of Maſter* Gunters *rare invention for projecting the Logarithmes in plane, I was adviſed by Maſter* Al'eaume *the Kings chiefe Engineir to dedicate my Booke to* MONSIEUR *the Kings brother, whoſe favourable admittance thereof, encouraged mee not long after to preſent him with this other Treatiſe alſo, which contayning the grounds and demonſtrations of all that was handled in the other Booke, he was alſo pleaſed graciouſly to accept. Thus expecting no leſſe favour at home, then I have* *already received among ſtrangers, I have now the ſecond time adventured to expoſe this ſhort Diſcourſe to the publike view.*

THE ADVERTISEMENT to the Reader.

A Treatiſe of this kind can hardly paſſe the preſſe without ſome eſcapes; wherefore the Author hath beene carefull to re-examine it ſince the Impreſſion, and hath cauſed the faults found in the diſcourſe to be placed immediately after it, and thoſe committed in the Tables to be likewiſe annexed unto them. Which few miſtakes being firſt amended, you may with confidence proceede to uſe the whole without danger of error.

THE CONSTRUCTION AND USE OF THE Logarithmeticall Tables.

CHAP. I.

The Definition, and Nature of Logarithmes in generall.

LOGARITHMES *are numbers, which being fitted to proportionall numbers retaine alwaies equall differences:*

So the *proportionall* numbers being 1,2,4,8,16,32,64, 128,256,512, 1024, &c. you may appoint unto them for *Logarithmes* the numbers ſubſcribed under the letters A, B, C, D, or any other at pleaſure; ſo that the *differences* of theſe Logarithmes be alwaies *equall*, when the *numbers*, unto which they are fitted, are *proportionall*.

Prop. nu.	A	B	C	D
1	1	5	5	35
2	2	6	8	32
4	3	7	11	29
8	4	8	14	26
16	5	9	17	23
32	6	10	20	20
64	7	11	23	17
128	8	12	26	14
256	9	13	29	11
512	10	14	32	8
1024	11	15	35	5
	Log.	*Log.*	*Log.*	*Log.*

But now to the end you may the more eaſily underſtand the *nature* of theſe Logarithmes, obſerve this *propoſition* following.

When of foure numbers given, the ſecond exceedes the firſt as much as the fourth exceedes the third;

third; the ſumme of the firſt and the fourth is equall to the ſumme of the ſecond and the third.

Example, theſe numbers 8, 14, 17, and 23 being given, the *ſumme* of 14, and 17 (being added together) is *equall* to the *ſumme* of 8, and 23.

From hence neceſſarily followes this *Corollary:*

When foure numbers are proportionall, the ſumme of the Logarithmes of the meane Numbers is equall to the ſumme of the Logarithmes of the extremes.

So the foure *proportionall* numbers being 2, 8, 16, and 64, and the Logarithme of 2, (in the columne of Logarithmes ſubſcribed under the letter C) being 8; The Logarithme of 8, 14; The Logarithme of 16, 17; And the Logarithme of 64, 23; I ſay, the ſumme of 14 and 17, the Logarithmes of the *meane* numbers, is equall to the ſumme of 8 and 23, the Logarithmes of the *extreme* numbers; as it appeares by the *example* of the propoſition aforegoing.

CHAP.

CHAP. II.

The Nature of the Logarithmes ſpoken of in the Treatiſe following.

THe numbers continually proportional, which M^r *Brigges* (in the Calculation of his *Chiliads*) hath propoſed to himſelf, are 1, 10, 100, 1000 &c. to which numbers hee hath aſſigned for Logarithmes ,0000, &c. 10000, &c. 20000, &c. 30000, &c. that is to ſay, to 1, the Logarithme 0000, &c. to 10, the Logarithme 10000, &c. to 100, the Logarithme 20000, &c. to 1000, the Logarithme 30000, &c. And after ſheweth the way how to make the *meane* Logarithmes, *viz.* of the numbers placed betwixt 1, and 10; betwixt 10, and 100; betwixt 100, and 1000, &c. ſome of which Logarithmes you ſhall finde placed in the *Tables* following;

I

A	B
1	0,000000
10	1,000000
100	2,000000
1000	3,000000
10000	4,000000
100000	5,000000
1000000	6,000000
10000000	7,000000

&c.

II

C	D
1	0,000000
2	0,301029
4	0,602059
8	0,903089
16	1,204119
32	1,505149
64	1,806179
128	2,107209
256	2,408239
512	2,709269
1024	3,010299

&c.

In the *firſt* of which you may obſerve (in the Columne marked by the letter A) a ranke of numbers continually *proportionall* from 1, and over againſt each number his reſpective *Logarithme* in the other Columne ſigned by the letter B.

In the *ſecond* table (in the Columne noted with the letter C) there is another ranke of numbers, which are alſo continually *proportionall* from 1, and juſt againſt each number his proper *Logarithme* (in the other Columne ſigned by the letter D) calculated according to the *reaſon* and proportion of the Logarithmes in the firſt *Table*; ſo that ſuppoſing the Logarithme of 1, to bee 0,000000, &c. and the Logarithme of 10 to be 1,000000, &c. the Logarithme of 2 will be found 0,301029 &c.

Now having put the Logarithme of 1 to be 0,000000, &c. and the Logarithme of 10 to be 1,000000, &c. from hence neceſſarily follow theſe *Corollaries.*

1. *The Characteriſtiques of the Logarithmes aſſigned to the numbers, propounded in the firſt of the Tables aforegoing, increaſe by unities.*

The *Characteriſtique* of a Logarithme is the firſt figure of the ſame Logarithme; ſo in the firſt of the premiſed *Tables* the *Characteriſtique* of 0,000000, (the Logarithme of 1) is 0; the *Charact.* of 1,000000, (the Logarithme of 10) is 1; the *Charact.* of 2,000000, (the Logarithme of 100) is 2, and ſo of the reſt in their order; And this *Characteriſtique* ought to be ſevered by a point or *comma* from the reſt of the Logarithme, as you may obſerve by the foreſaid *tables:*

2. *The Characteriſtique of the Logarithme of any number comprehended betwixt any two of the proportionals in the firſt Table, differs not from the Characteriſtique of the Logarithme of the firſt of thoſe two proportionalls.*

So in the ſecond *table* aforegoing you may obſerve, that 2, 4, and 8 being numbers comprehended betwixt 1 and 10, the *common Characteriſtique* of their Logarithmes is 0; *viz.* the *Charact.* of 0,000000 the Logarithme of 1;

of 1; Againe *16*, *32*, and *64*, being numbers conteined betwixt 10 and 100, the *Charact.* of their Logarithme is 1, *viz.* the *Charact.* of 1,000000, the Logarithmes of 10; In like manner the *Charact.* of the Logarithmes of 128, 256, and 512 (being numbers ſcituate betwixt 100 and 1000) is 2, which is alſo the *Charact.* of 2,000000 the Logarithme of 100; and ſo conſequently of all other numbers comprehended betweene thoſe *proportionall* numbers of the *table* A.B.

3. *The Characteriſtique of any Logarithme is alwaies an unit leſſe, then the number of places, whereof the number, unto which it belongs, doth conſiſt*: For if a Logarithme be propounded, which hath for his *Characteriſtique* 0, the number, unto which that Logarithme appertaines (by the laſt *Corollary*) exceeds not 10, and therefore muſt needs conſiſts but of *one* onely place; So in the *table* C D 0,903089 (whoſe *Charact.* is 0) is the Logarithme of 8, which is a number that conſiſts of *one* place; in like manner when a Logarithme is propounded, which hath for his *Charact.* 1, the number, unto which that Logarithme belongs, conſiſts of *two* places: ſo againe (in the *table* C D) 1,806179 (whoſe *Charact.* is 1) is the Logarithme of *64*, which therefore

conſiſts of *two* places; and ſo conſequently of the reſt.

4. *The Logarithmes of this kind ought all to conſiſt of equall places*: For you may propound them to conſiſt either of *ſeven* places, as thoſe in the premiſed *tables* A B and C D, or of *fifteene* places, as M^r *Brigges* hath thought moſt convenient (principally for large computations) or of as many places as you pleaſe, but when you are once reſolved of how many places the Logarithmes of your *table* ſhall conſiſt, you muſt not alter your firſt reſolution, as to make the Logarithme of 2 to be 301029, *viz.* to conſiſt of *ſix* places, & the Logarithme of 16, *viz.* 1, 204119 to have *ſeven*, but rather in this caſe you are to prefix before 30102 a *cypher* to make it conſiſt alſo of *ſeven* places, and then the compleate Logarithme of 2 will bee 0, 301029, as in the *table* C D, which 0 ſerves likewiſe for the *Charact.* of that Logarithme, as before hath bene ſhewed.

5. *In multiplication, the ſumme of the Logarithmes of the multiplicand and of the multiplicator is equall to the Logarithme of the Product.*

For as much as in every multiplication there are ſome *proportionall* numbers, *viz.* as 1 is to the multiplicand, ſo is the multiplicator to the Product, and the Logarithme of 1 being 0, 000000, it is manifeſt by the *Corollary* of the laſt Chapter, that the *ſumme* of the Logarithmes of the Multiplicand and Multiplicator is equall to the *Logarithme* of the product; For *example*, 16 being given to be multiplied by 4, the product is 64, and here the *proportionall* numbers are 1, 16, 4, and 64 (for as 1 to 16, ſo is 4 to 64) I ſay then that the *ſumme* of the Logarithmes of 16 and 4 (the two *meane* numbers of that propoition) is equall to the *ſumme* of the Logarithmes of 1 and 64 (the two *extremes*) by the *Corollary* laſt cited; but the Logarithme of 1 being 0, 000000 the *addition* thereof alters not the Logarithme of 64, therefore the *Logarithme* of 64 the product, muſt needes bee equall to the *ſumme* of the Logarithmes of 16 and 4, the termes propounded to be multiplied: For better explanation of this *Corollary* find in the premiſed *table* C D the Logarithme of 16, which is 1, 204119, as alſo the Logarithme of 4, being 0, 602059, theſe Logarithmes if you add together, their *ſumme* is 1, 806178, which is the Logarithme of 64 (the product) as you may obſerve by the ſame *table*: onely here there wants an unit in the laſt place, but the want of an unit or two in the laſt figure of the Logarithme either in this or any other operation whatſoever begetteth no error in the worke.

6. *In diviſion, the ſumme of the Logarithmes of the diviſor and of the quotient is equall to the Logarithme of the dividend:* For as the diviſor is to 1, ſo is the dividend to the quotient; and therefore (1 being alwaies in *diviſion* one of the *meane* numbers of that proportion) I ſay, the Logarithme of the dividend, notwithſtanding the *Addition* of 0, 000000 (the Logarithme of 1) unto it, remaines ſtill the ſame without alteration: For *example*, 64 being given to bee divided by 4, the quotient wilbe 16, and the *ſumme* of the Logarithmes of 4 (the *diviſor*) and 16 (the *quotient*) is equall to the *Logarithme* of 64 (the *dividend*) as appeares by the *example* of the laſt Corollary.

7. *In any ranke of numbers continually proportionall from 1, the Logarithme of any one of them being divided by the denomination of the power, which it challengeth in the ſame ranke, the quotient will give you the Logarithme of the roote.*

In the ranke of the *proportionall* numbers of the *table* C D, 2 being the *roote* or firſt power, 4 the *ſquare* or ſecond power, 8 the *Cube* or third povver, 16 the *biquadrate* or fourth power, 32 the *fiſt* power, 64 the *ſixt* power, &c. I ſay the Logarithme of 4,8,16,32,64, or of any of the other ſubſequent *proportionalls* in that ranke being divided by the *Denomination* of the povver, that the ſame *proportionall* claimeth in the ſame ranke, you ſhall finde in the quotient the Logarithme of 2 the *roote*; For *example*, in the ſame *table* the Logarithme of 4 (the *ſquare* or ſecond povver) *viz.* 0,602059 being given, I demaund the Logarithme of 2 the *roote*; here the *denomination* of the povver, that the *proportionall* 4 challengeth in that ranke (being the *ſquare* or ſecond povver) is 2, wherefore if 0,602059 the Logarithme of 4 bee divided by 2, the quotient will be 0,301029, which is the Logarithme of 2, the *roote*, as appeares by the ſame Table: So likewiſe 0,903089, the Logarithme of 8 (the *Cube* or third povver) being divided by 3, leaves you in the quotient the ſame 0,301029; And 3,010299 the Logarithme of 1024 (the *tenth* povver) being propounded and divided by 10 (the *denomination* of his povver) gives you in the quotient 301029, before which if you prefixe 0 for the *Characteriſtique* (according to the fourth *Corollary* of this chapter) the totall is 0,301029, *viz.* the Logarithme of 2 the *roote*, as before; and ſo conſequently of the reſt.

8. *In any ranke of numbers continually proportionall from* 1, *the Logarithme of the roote being multiplied by the denomination of any of the powers, the product is the Logarithme of the ſame power:* This *Corollary* is the *inverſe* of the laſt: For *example*, in the ranke of numbers propounded in the laſt *Corollary*, 0,301029 (the Logarithme of 2 the *roote*) being *doubled* or multiplied by 2 produceeth 0,602059, the Logarithme of 4, the *ſquare*, or ſecond power, and the ſame Logarithme 0,301029 being *trebled* or multiplied by 3 produceeth 0,903089 the Logarithme of 8 the *Cube*, or third power, and ſo of the reſt. The *truth* of theſe two laſt *Corollaries* may be evidently demonſtrated by the *definition* of Logarithmes, being conſidered together with the 12 *Rule* of the 9 *chapter* of the firſt booke of my *Arithmetique*.

CHAP.

CHAP. III.

The Geneſis or Fabrique of the Logarithmes treated of in the inſuing diſcourſe.

THE Logarithme of 1, being put 0,000000, the Logarithme of 10, 1, 000000, the Logarithme of 100, 2, 000000, &c. in the next place it will be requiſite to ſhew the way how to make the Logarithmes of the *meane* numbers, *viz.* Of 2,3,4, &c. with are ſituate betwixt 1, and 10; Of 11,12,13, &c. which are placed betwixt 10, and 100; and ſo of the reſt, which wee intend to explaine in the reſolution of the *Problemes* following.

PROBL. I.

The number* 10, *being given, to finde ſo many continuall meanes betwixt

it and 1, till that continuall meane, which comes neereſt 1, may be a mixt number leſſe then 2, and ſo neer 1, that it may have ſeaven Cyphers placed before the ſignificant figures of the numerator.

ALthough amongſt the *proportionall* numbers of the Table AB, I might make choiſe of any one of them to effect our preſent deſigne, yet becauſe the number 10, being the leaſt, is the fitteſt for that pupoſe, I nominate that to be taken rather then any other; *Having therefore placed after the number* 10, *towards the right hand a competent company of Cyphers* (viz. *eight and twenty to the end the operation may be thereby made the more exact) extract the Square roote of that number ſo enlarged, this done, you ſhall find that roote to be*

$$3\frac{16227766016837}{100000000000000}$$

Which being a mixt number conſiſting of three *Integers* and a *Decimall fraction*, may likewiſe (according to the Rules of *decimal Arithmetique*) be more conveniently expreſt without the denominator; thus,

3.16227766016837.

Havin

Having by this meanes found the exact root of 10, *annex unto that root ſo found fourteen cyphers more, and then working by that intire number ſo ordered, as if it were a whole number, extract the ſquare root thereof,* which you ſhall finde to be

1.77827941003892.

And ſo proceeding ſucceſſively by a continued extraction you are to produce ſeven and twentie ſquare roots, or continuall meanes betwixt 10 *and* 1, *viz.* ſuch as you finde placed in the *firſt Columne* of the table hereunto annexed; which done, you may obſerve, that the three laſt *continuall meanes* (marked by the letters *g*, *h*, and *l*) *viz.*

1.00000006862238
1.00000003431119
1.00000001715559

are each of them *mixt numbers* leſſe then 2, and greater than 1, and likewiſe to have *ſeven cyphers* placed before the *ſignificant figures* of their *Numerators*, according to the demand of this preſent *Probleme*.

	10,0000, &c.	1,0000000000000000	
A	3.16227766016837	0,5000000000000000	D
B	1.77827941003892	0,2500000000000000	
C	1,33352143216332	0,1250000000000000	
	1.15478198468945	0,0625000000000000	
	1.07460782832131	0,3125000000000000	
	1.03663292843769	0,0156250000000000	
	1,01815172171818	0,0078125000000000	
	1.00903504484144	0,0039062500000000	
	1.00450736425446	0,0019531250000000	
	1.00225114829291	0,0009765625000000	
	1.00112494139987	0,000488281250000	
	1.00056231260220	0,000244140625000	
	1,00028111678778	0,000122070312500	
	1.00014054851694	0,000061035156250	
	1.00007027178941	0,000030517578125	
	1,00003513527746	0,0000152587890625	
	1.00001756748442	0,0000076293945312	
	1.00000878270363	0,0000038146972656	
	1.00000439184217	0,0000019073486328	
	1.00000219591867	0,0000009536743164	
	1.00000109795873	0,0000004768371582	
	1.000000054897921	0,0000002384185791	
	1.0000000274448957	0,0000001192092895	
	1.0000000137244770	0,0000000596046447	
G	1.0000000068622238	0,0000000298023223	N
H	1.0000000034311119	0,0000000149011611	K
L	1.0000000017155559	0,0000000074505805	M

Afte

After the ſame maner may you finde out as many *continuall meanes*, as you pleaſe, betwixt any other number given and 1, if you joyne *Cyphers* thereunto, as you have beene directed to annex them unto the number 10: Neverthelesſe here *obſerve*, that although (by the reſolution of this *Probleme*) you are adviſed to extract ſo many *continuall meanes*, that the laſt ſhould have but *ſeven cyphers* before the ſignificant figures of his numerator; yet you are to underſtand, that this is onely neceſſary when you intend that the Logarithmes of the *Table*, you are to make, ſhould conſiſt of *ſeven places*, as thoſe in the premiſed *Tables* A B, and C D; For when you intend the Logarithmes of your *Table* ſhall conſiſt of *eight, ten, twelve, fifteen*, or of any other greater number of figures, it will be requiſite to produce ſo many *Continuall meanes*, till the laſt of them may have as many *cyphers*, before the ſignificant figures of his numerator, as the Logarithmes of your intended *table* ſhall have *places*.

PROBL. II.

The Logarithme of* 10, *being given to make the Logarithmes of all the continuall

tinuall meanes contained betwixt 10. *and* 1.

THis Probleme may eaſily be reſolved by the ſeventh Corollarie of the laſt chapter; for (in the Table aforegoing) the number a *being the ſquare root of* 10, *if you divide the Logarithme of* 10 *by* 2, *you have the Logarithme of the ſaid number* a: *in like manner dividing the Logarithme of the number* a *by* 2, *you have the Logarithme of the number* b, *and ſo proceeding in the ſame order, the Logarithmes of all the continuall meanes betwixt* 10 *and* 1 *may be eaſily found out.*

For *example*, in the ſecond Columne of the *Table* aforegoing 1,000000000000000 being aſſigned the Logarithme of 10, halfe of it, *viz.* 0,50000, &c. ſigned by the letter *d*, is the Logarithme of the number *a* (the *ſquare root* of 10) by the *Corollarie* laſt cited: In like manner 0,25000, &c. being halfe 0,50000, &c. is the Logarithme of the number *b*, and 0,125000, &c. the Logarithme of the number *c*, and ſo conſequently of the reſt; ſo that at laſt, as you have in the firſt Columne of the laſt *Table* 27 *continuall meanes* betwixt 10 and 1, as aforeſaid; ſo may you likewiſe make (by the reſolution of this *Probleme*) for all thoſe *continuall meanes*, reſpective Logarithmes, as you may obſerve them placed in the other Columne of the ſame *Table.*

PROBL. III.

A mixt number being given, which being leſſe then 2, and greater then 1, comes ſo neere unto 1, that it hath ſeven ciphers placed before the ſignificant figures of the Numerator, to make his proper Logarithme.

BEfore we come to the reſolution of this *Probleme*, this *Propoſition* following, muſt firſt be obſerved;

when a mixt number, which being leſſe then 2, and greater then 1, comes ſo neere to one, that it hath ſeven cyphers placed before the ſignificant figures of the Numerator; the firſt ſeven ſignificant figures of his Numerator are double the firſt ſeven ſignificant figures of the Numerator of his ſquare root.

For *example*, the number 1.000000068622-38562102573718748 2, being propounded, which being a *mixt* number leſſe then 2, and greater then 1, hath *ſeven* ciphers placed before the ſignificant figures of his Numerator.

I *ſay*,

I *ſay*, 6882238, the firſt *ſeven* ſignificant figures of his Numerator are *double* to 3431119 the firſt *ſeven* ſignificant figures of the Numerator of the number 1.000000034311192, which is the *ſquare root* of the ſame number ſo propounded: And this obſervation holds likewiſe true in all other numbers, of the ſame kind, though they have moe or fewer cyphers than *ſeven* placed before the ſignificant figures of their Numerators; for look *how many* cyphers are ſo placed before the ſignificant figures of their Numerators, *ſo many* of their firſt ſignificant figures are double to *as many* of the firſt ſignificant figures of their *ſquare roots*: Howbeit, the reaſon, why in this propoſition I confine my ſelfe againe to the number of *ſeven* figures, is, becauſe *ſo many* onely are conſiderable for the making of the enſuing *Tables*, whoſe Logarithmes I intend ſhall conſiſt of *ſeven* places, according to the *reaſon* formerly alleaged in the firſt *Problem* of this Chapter.

From hence follow theſe Corollaries:

1. *The ſignificant figures of the Numerator of a number of this kinde, and the ſignificant figures of the Numerator of his ſquare root leſſen themſelves like their Logarithmes, that is to ſay, by halfes.*

For *example*, in the lower end of the *Table* aforegoing, as the Logarithme *k* is *halfe* the Logarithme *n*; ſo 3431119, the ſignificant figures of the Numerator of the number *h*, are the *halfe* of 6862238, the ſignificant figures of the Numerator of the number *g*.

2. *Therefore any two numbers of this kinde being given, their Logarithmes, and the ſignificant figures of their Numerators are proportionall.*

Example, the numbers of *g* and *h* being given, I ſay, as 6862238, the *ſignificant figures* of the Numerator of the number *g*, are to 3431119, the *ſignificant figures* of the Numerator of the number *h*; ſo is *n*, the *Logarithme* of the ſaid number *g*, to *k*, the *Logarithme* of the ſaid number *h*: In like manner the numbers *g* and *l* being given, as 6862238 is to 1715559, ſo is the *Logarithme n* to the *Logarithme m*.

And this Rule holds true in any other number of this kinde, though it be not one of the *continuall meanes* betwixt 10 and 1: for the *ſignificant figures* of the Numerator of any ſuch number beare the ſame *proportion* to his proper *Logarithme*, that the *ſignificant figures* of any of the numbers marked by the letters *g*, *h* or *l* beare to *his*.

Theſe things being thus cleared, it is manifeſt, that a number of this *kinde* being given, the Logarithme thereof may be found by the *Golden Rule:* For,

As the ſignificant figures of the Numerator of any one of the numbers (ſigned in the firſt Columne of the laſt Table by the letters g, h *or* l) *are to his reſpective Logarithme:*

So are the ſignificant figures of the Numerator of the number given, to the Logarithme of the ſame number.

Example, the number 1.00000001021301 being given, I demand the Logarithme thereof: I ſay then,

As 6862238 the *ſignificant figures* of the Numerator of the number *g*, are to 29802322, the *Logarithme* of the ſame number *g*.

So are 1021301, the *ſignificant figures* of the Numerator of the number given, to 4357281, the *Logarithme required*.

Before which if you prefix nine ciphers, to the intent it may have as many places as the Logarithmes in the laſt premiſed *Table*, (*viz*. 16.) according to the fourth *Corollarie* of the laſt *Chapter*, the true & intire Logarithme of 1,00000001021301, the nomber given, is 0,0000000004357281; and thus at laſt you have the cleare reſolution of the *Probleme* propounded.

PROBL.

PROBL. IV.

Any number whatſoever being given, to make the Logarithme of the ſame number.

YOu may make the Logarithme of any number whatſoever, if firſt you produce ſo many continuall meanes betwixt the ſame number and 1, *till the continuall meane, which comes neereſt* 1 *may have ſeven ciphers placed before the ſignificant figures of his Numerator; For, this being done, you may readily finde out the Logarithme of that continuall meane; and then by often doubling, and redoubling that Logarithme ſo found (according to the number of the continuall meanes at firſt produced) at laſt you ſhall fall upon the Logarithme of the number given.*

Example, the number 2 being given, I demand the Logarithme thereof: Here firſt (according to that which hath formerly been taught in the firſt *Probleme* of this Chapter) I produce ſo many *continuall meanes* betwixt 2 and 1, till that which comes neereſt 1 hath *ſeven* ciphers placed before the ſignificant figures of the Numerator, which after *three* and *twenty* continued extractions I finde to be 1.000000008262958; This *continuall meane* being thus found, (by the direction of the *Probleme* aforegoing) I finde the Logarithme thereof, which is 0,0000000035885571 (For

As 6862238 *to* 29802322,
So 8262958 *to* 35885571)

This Logarithme being *doubled*, will produce (by the laſt *Corollarie* of the ſecond *Chapter*) the Logarithme of the *continuall meane* next above 1.000000008262958; and ſo if you *double* ſucceſſively the Logarithme of each *continuall meane*, one after another, according to the number of the extractions (*viz.* *three* and *twenty* times in all) at laſt you ſhall happen upon the Logarithme 0,301029987975168, which is the Logarithme of 2, the number propounded: The whole *frame* of the work is plainly ſet down in the *Table* following; For in the firſt Columne thereof you have three and twenty *continuall meanes* betwixt 2 and 1, and in the other Columne you ſhall finde the reſpective Logarithmes, which are produced by a continued *doubling* and *redoubling* of 0,0000000035885571 the Logarithme of 1.000000008262958, the laſt *continuall meane* in the *Table*.

2.0000,&c.

2,0000,&c.	0,301029987975168
1. 41421356237309	0,150514993987584
1. 18920711500272	0,075257496993792
1. 19050713266525	0,037628748496896
1. 04427378243220	0,018814374248448
1. 02189714865645	0,009407187124224
1. 01088928605285	0,004703593562112
1. 00542990111387	0,002351796781056
1. 00271127505073	0,001175898390528
1. 00135471989237	0,000587949195264
1. 00067713069319	0,000293974597632
1. 00033850805274	0,000146987298816
1. 00016923970533	0,000073493649408
1. 00008461627271	0,000036746824704
1. 00004230724140	0,000018373412352
1. 00002115339696	0,000009186706176
1. 00001057664255	0,000004593353088
1. 00000528830729	0,000002296676544
1. 00000264415015	0,000001148338272
1. 00000132207420	0,000000574169136
1. 00000066103688	0,000000287084568
1. 00000033051838	0,000000143542284
1. 00000016525917	0,000000071771142
1. 00000008262958	0,000000035885571

But now (becauſe the Logarithmes of our intended *Tables* ſhall conſiſt onely of *ſeven* places, (according to the *obſervation* made in the end of the firſt *Probleme* of this *Chapter*) Of the Logarithme 0, 30102998797 5·68 I take onely the firſt *ſeven* figures, towards the left hand, adding 1 unto the ſeventh figure, becauſe the eight figure (being 9) doth almoſt carry the value of an unite to the ſame ſeventh figure; and then at laſt the preciſe Logarithme of 2, the number given, will bee found 0,301030: And thus as the Logarithme of 2 is made, ſo may you likewiſe make the Logarithme of any other number whatſoever; Howbeit the Logarithmes of ſome few of the *prime* numbers being once thus diſcovered, the Logarithmes of many other *derivative* numbers may be found out afterwards without the trouble of ſo many continued *extractions* of the ſquare root. For *example*, having thus made the Logarithme of 2, you may eaſily know the Logarithme of 5; for dividing 10 by 2, the quotient will be 5; but the *ſumme* of the Logarithmes of the diviſor, and the quotient, is equall to the *Logarithme* of the dividend (by the ſixt *Corollarie* of the ſecond Chapter.) And therefore if you ſubſtract 0,301030 (the Logarithme of 2) from 1, 000000 (the Logarithme of 10) the remainder (*viz.* 0, *698970*) is the Logarithme of 5.

Againe, beſides the Logarithme of 5, with like *facility* you may finde the Logarithme of any other number, that is made by the multiplication or diviſion of theſe three numbers 2, 5, and 10, *viz.* Of the numbers 4, 8, 16, 32, 64, &c. Of the numbers 25, 125, 625, &c. Of the numbers 20, 50, 100, 200, &c. Following the direction given you in the foure laſt *Corollaries* of the ſecond Chapter.

After this manner is that *Table* calculated, which you ſhall finde in the firſt Page of the *Tables* placed after this diſcourſe, which you may obſerve to conſiſt of ſix Columnes, whereof the firſt, third, and fift, ſigned by the letter N.) containe all abſolute numbers from 1. to 100; and in the ſecond, fourth, and ſixt, (entituled Logar.) is placed juſt againſt each number his reſpective Logarithme: So in the ſecond Columne of the ſame *Table*, 0, 477121 is the Logarithme of 3; In the fourth, 1, 556302 the Logarithme of 36; and in the ſixt Columne 1, 838849 is the Logarithme of 69, &c.

And this *Table* may ſerve to ſhew you, how the *Tables* of *Logarithmes* (though never ſo large)

large) are uſually ordered; for *regularly* all the abſolute numbers ought to be ſet downe at large, and juſt againſt each number (in the next *Columne*) ought to bee placed his reſpective Logarithme, as you may obſerve them in the ſame *Table*: But now if I (intending in this Edition to preſent to the *publique* the Logarithmes of all abſolute numbers from 1 to 400000) ſhould obſerve the ſame order, that *Table* would too much incumber this little Volume which I intend portable: And therefore I have invented another *Table*, which immediately followeth the foreſaid Table of 99 Logarithmes, and is intituled (*The Table of Logarithmes*) wherein the Logarithmes of all abſolute numbers under 10000 are expreſly ſet downe, and by help whereof the Logarithmes of all *abſolute* numbers under 400000 may be alſo readily diſcovered, as ſhall more plainly appeare hereafter.

And now for the *deſcription* of this laſt mentioned *Table*, you may obſerve, that it conſiſts of twelve *Columnes*, *viz.* ſix in the *left* hand Pages, and other ſix in the *right* hand Pages; of theſe twelve *Columnes*, the firſt (noted at the top of every Page by the letter *N*) containes all the *abſolute* numbers from 1 to 1000, as they ſtand one after another in their *naturall* order. Againe, the ten *Columnes* (intituled by the figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9) containe Logarithmes without their *Characteriſtiques*, or firſt figures. And laſtly, in the twelfth *Columne* (marked by the letter *D*) are ſet downe the *common differences* of thoſe Logarithmes. And note, that the *Characteriſtiques* of Logarithmes are quite left out of theſe *Tables*, becauſe *they* without the *Tables* may be readily diſcovered, and the *Tables* being expreſt without *them*, are thereby made more apt for *generall* uſe, as you ſhall be more plainly taught hereafter.

As for the other *Table*, which followes this *Table* of *Logarithmes* (intituled *Artificiall Sines and Tangents*) it is no other than an ordinary *Table* of *Artificiall Sines* and *Tangents*, which are the Logarithmes of the right *Sines* and *Tangents*, ſet forth by *Rheticus*, *Petiſcus*, and others: Of this I need make no other *deſcription*, than onely to name it, becauſe the *diſpoſition* thereof differs not from the *order* of other *Tables* of the ſame kinde.

CHAP. IV.

The uſe of the Logarithmeticall Tables in finding out the Logarithme of any number, and the number of any Logarithme propounded.

THus farre the *conſtruction* of the *Logarithmeticall Tables*, now come we to their *uſe*, which is twofold:

1. *To finde the Logarithme of any number, or the number of any Logarithme given.*

2. *To reſolve divers neceſſary Problemes in Arithmetique and Geometrie.*

As for the *firſt* of theſe, you ſhall finde *it* explained in theſe *Problemes* following.

PROBL.

PROBL. I.

A whole number under 100 *being given, to finde the Logarithme thereof.*

T*Urne to the Table of ſix Columnes (which you ſhall finde upon the firſt Page of the inſuing Tables) this done, having found the number given in the firſt, third, or fift Columne of that Table, juſt againſt that number in the next Columne towards the right hand you ſhall finde the Logarithme required.*

So in that *Table*, the Logarithme of 20 will be found 1,301030, and the Logarithme of 75 is 1,875061,&c.

PROBL. II.

A whole number that conſiſts of three places being given, to finde the Logarithme thereof.

F*Inde the number propounded in the firſt Columne of the Table, intituled the Table of Logarithmes (ſigned with the letter* N*) this done,*

done, juſt againſt that number in the next Columne, (ſigned by o *) you ſhall finde a number conſiſting of ſix figures, before which if you prefix a Characteriſtique, correſpondent to the number propounded, that intire number ſo ordered is the Logarithme required.*

Example, 125 being given, I demand the Logarithme thereof: The number 125 I finde in the firſt Columne of the *Table of Logarithmes,* upon the firſt Page of the ſame *Table,* and juſt againſt it in the next Columne (ſigned by *0*) I finde 096910; now if before 096910 I prefix 2, for the *Characteriſtique* (becauſe the number given conſiſts of *three* figures or places, according to the ſecond *Corollarie* of the ſecond Chapter aforegoing) the Logarithme of 125 the number propounded will be found 2,096910: So likewiſe (upon the third Page of the ſame Table) you ſhall finde 2,176091 to be the Logarithme of 150, and (Page 5) 2,267172 to be the Logarithme of 185, &c.

PROBL.

PROBL. III.

A number that conſiſteth of foure places being given, to finde the Logarithme thereof.

Find the three firſt figures of the number given in the firſt Columne of the Table of Logarithmes, as before; then ſearch the laſt figure thereof amongſt the figures, placed in the top of the Pages, where you have ſo found thoſe three firſt figures: this done, juſt againſt thoſe three firſt figures, in the Columne under the laſt figure, you ſhall finde a number, before which if you prefix a proper Characteriſtique for the number given, that intire number ſo ordered is the Logarithme you looke for.

Example, 1257 being propounded, I demand the Logarithme thereof; 125, the *three firſt* figures of the number given, I finde (as before) in the firſt Columne of the *Table of Logarithmes,* and likewiſe 7, the *laſt* figure thereof, upon the next Page in the top of the ninth Columne of the ſame *Table:* this done, in the angle of *Poſition* (juſt againſt 125, and under the figure 7) I obſerve this number 099335, 099335, before which having placed 3 for the correſpondent *Characteriſtique* of 1257, (being a number of *foure* places) at laſt I produce 3,099335, which is the true Logarithme of the ſaid 1257, the number propounded. In like manner 3,096910 is the Logarithme of 1250, and 3,636989 the Logarithme of 4335, &c.

PROBL. IV.

A number that conſiſteth of five places being given, to finde the Logarithme thereof.

Find the Logarithme of the foure firſt figures thereof (beſides the Characteriſtique) by the Probleme aforegoing; this done, obſerving the common difference (in the twelfth and laſt Columne of the Table, juſt againſt the three firſt figures of the number given, found in the firſt Columne) multiply that difference by the laſt figure of the number given, and cutting off one figure from the product towards the right hand, adde the reſt thereof to the Logarithme found: laſt of all, if you prefix before that Logarithme ſo found the proper Characteriſtique of the number given, that Logarithme ſo ordered is the Logarithme required.

Example, 12572 being propounded, I demand the Logarithme thereof; by the *Probleme* aforegoing the Logarithme of 1257 (beſides the *Characteriſtique*) is 099335, and the *common difference* in the laſt Columne of the *Table* (juſt againſt 125, the *three firſt* figures of the number given) is 346, which being multiplied by 2 (the laſt figure of the number given) the *product* is 692; wherefore if I adde 69 to 099335, and likewiſe prefix before that ſumme, 4 (the proper *Characteriſtique* of the number given) at laſt I ſhall produce 4,099404, which is the Logarithme of the number propounded. So 4,099335 is the Logarithme of 12570, and 4,911205 the Logarithme of 81509, &c.

PROBL. V.

A number that conſiſts of ſix places (under 400000) being given, to finde the Logarithme thereof.

Finde the Logarithme of the firſt foure figures thereof, (beſides the Characteriſtique) as before; This done, multiply likewiſe, as before, the common difference by the two remaining figures of number given; then cutting off two figures from *the product towards the right hand, if you adde the reſt thereof to the Logarithme found, and prefix, as before, a proper Characteriſtique, that Logarithme ſo ordered is the ſame you looke for.*

Example, let 125724 be the number propounded, and let the Logarithme thereof be required: The Logarithme of 1257 (beſides the *Characteriſtique*) is 099335, and the *common difference* is 346, which if you multiply by 24 (the two laſt figures of the number given) the *product* will be 8304: Now therefore if I adde 83 to 099335, their *Aggregate* is 099418, before which if I prefix 5 (the reſpective *Characteriſtique* of the number given) at laſt I produce 5, 099418 for the correſpondent Logarithme of the number propounded.

PROBL. VI.

A fraction being given to finde the Lorithme thereof.

DEduct the Logarithme of the Numerator out of the Logarithme of the Denominator; This done, the remainder is the Logarithme of the fraction propounded.

Example, $\frac{3}{4}$ being propounded, the Logarithme thereof is 0,124938; for (by the firſt *Probleme* of this Chapter) the Logarithme of 4 is 0,602060, out of which if I ſubſtract 0,477121 the Logarithme of 3, the remainder is 0,124939, the Logarithme of $\frac{3}{4}$ the number propounded. So likewiſe 1,002425 is the Logarithme of $\frac{125}{1257}$ and 2,002494 the Logarithme of $\frac{125}{12572}$ &c. as appeares by the work following.

125	2,096910	125	2,096910
1257	3,099335	12572	4,099404
	1,002425		2,002494

But here obſerve, that the Logarithme of a *Fraction*, or broken number is alwayes *defective*, that is, the value thereof is alwayes leſſe then 0, or *nothing*, for the Logarithme of 1 being put 0,000000, the Logarithme of $\frac{3}{4}$, which is leſſe than 1, muſt needs be leſſe than *nothing*; and by how much neerer a *Fraction* approaches to 1, by ſo much leſſe is the *quantity* of his Logarithme, *& è contra:* becauſe as the Logarithmes of numbers, that are *greater* than 1, increaſe *à nihilo ad infinitum* above 1; ſo the Logarithmes of numbers *leſſe* than 1, increaſe likewiſe *a nihilo ad infinitum* under 1: when therefore you meet with a Logarithme of this kinde, to diſtinguiſh it from a perfect Logarithme, prefix before it this marke ——, as in the *Examples* before premiſed. See my *Arithmetique Lib.* 2. *Cap.* 6. *R.* 2.

In like manner theſe *Decimall Fractions* $\frac{25}{100}$ and $\frac{05}{100}$ being propounded (which likewiſe may be written without their *Denominators*, thus .25, and thus .05) I ſay, their Logarithmes are —— 0,602060, and —— 1,301030, as may appeare by the worke following;

25	1,397940	5	0,698970
100	2,000000	100	2,000000
	—— 0,602060		—— 1,301030

Howbeit, when the Fraction propounded is a Decimall, you may more eaſily finde the Logarithme thereof, thus; Firſt finde the reſt of the Logarithme of the Decimall given (beſides the Characteriſtique) as if it were a whole number; then taking the complement which that Logarithme wants of the intire Logarithme of 10, *if you place before that complement his proper Characteriſtique (which*

(which ought to conſiſt of ſo many units, as the Decimall given hath cyphers prefixt before it) that complement ſo ordered is the Logarithme required.

Example, the *Decimall* .125 being given, I demand the Logarithme thereof; The Logarithme of 125 beſides the *Characteriſtique* (by the ſecond *Probleme* of this Chapter) is 096910, whoſe *complement* to the Logarithme of 10 is 903090 (for if you ſubſtract 096910 out of 1,000000, the *intire* Logarithme of 10, the remainder is 903090;) now therefore if before the *complement* 903090 you place the *Characteriſtique* 0 (becauſe the *Decimall* given hath no cyphers prefixt before it) the *intire* Logarithme of .125 will be found—0,903090: So likewiſe, the Logarithme of .0125 is—1,903090, and the Logarithme of 00125 is—2,903090, &c.

PROBL. VII.

A mixt number being given, to finde the Logarithme thereof.

REduce the mixt number into an improper Fraction; this done, if you ſubſtract the

Logarithme of the Denominator out of the Logarithme of the Numerator, that which remaines is the Logarithme required.

Example. The mixt number 125 $\frac{3}{4}$ being propounded, I demand the Logarithme thereof; That number being reduced to an improper *Fraction*, is $\frac{503}{4}$ likewiſe the Logarithme of 503, the *Numerator* (by the ſecond *Probleme* of this Chapter) is 2,710568, and the Logarithme of 4, the *Denominator* (by the firſt *Probleme* of this Chapter) is 0,602060: wherefore if I deduct 0,602060 the Logarithme of 4 out of 2,710568 the Logarithme of 503, the remainder *viz.* 2,108508 is the Logarithme of 125 $\frac{3}{4}$ the number propounded.

Here when the Fraction annexed is a Decimall, you may likewiſe finde the Logarithme thereof, thus; Conceiving the number given to be a whole number, finde the reſt of the Logarithme thereof (beſides the Characteriſtique) by the 1,2,3,4, or 5 Problemes aforegoing; this done, if you place before that Logarithme ſo found, his proper Characteriſtique (viz. a figure conſiſting of ſo many units, wanting one, as the whole part of the number given conſiſts of places) that intire Logarithme ſo ordered is the Logarithme required.

Example. The number $125\frac{72}{100}$ (which likewiſe may be expreſſed without the *Denominator*, thus, 125.72, or thus, 125|72) being given, I demand the Logarithme thereof; the Logarithme of 12572 (beſides the *Characteriſtique*) is 099404 (by the fourth *Probleme* of this Chapter) before which if I prefix 2, (becauſe 125, the *whole part* of the number given, conſiſts onely of *three* places) that *intire* Logarithme ſo ordered is 2,099404, which is the Logarithme of 125|72 the number propounded.

You may perceive by that which hath been delivered in theſe two laſt *Problemes*, that when the Fractions propounded are *Decimals*, their Logarithmes are more eaſily found, than when they are other Fractions; whereupon I have cauſed the inſuing *Table* to be hereunto annexed, which may ſerve for the Reduction of *compound* Fractions, *viz.* of the Fractions of *Coine*, *Weights*, *Meaſures*, *Time*, and *Dozens*) to *Decimals*, to the end that when any ſuch termes come in queſtion, their Logarithms may be the more readily diſcovered: Howbeit here (leſt I ſhould too much inlarge this little Volume) I intend onely to preſent the *Table* it ſelfe, referring thoſe that deſire to be better informed concerning it, to the eleventh Chapter of the firſt Booke of my Arithmetique.

The Table of Reduction.

Engliſh Money.

19	95	11	55	3	15	6	025
18	9	10	5	2	1	5	020833
17	85	9	45	1	05	4	016667
16	5	8	4	11	04583	3	0125
15	75	7	35	10	041667	2	a83333
14	7	6	3	9	0375	1	a41667
13	65	5	25	8	033333	3	a3125
12	6	4	2	7	029167	2	a20833
						1	a10417

Troy Weight.

11	91667	16	066667	3	0125	13	a22569
10	83333	15	9625	2	a83333	12	a20833
9	75	14	058333	1	a41667	11	a19097
8	66667	13	054167	23	a39930	10	a17361
7	58333	12	05	52	a38194	9	a15625
6	5	11	045833	21	a36458	8	a13889
5	41667	10	041667	20	a34772	7	a12153
4	33333	9	0375	19	a32986	6	a10417
3	25	8	033333	18	a3125	5	e 86806
2	16667	7	021967	17	a29514	4	e 69444
1	083333	6	025	16	a27778	3	e 52083
19	079167	5	020833	15	a26642	2	e 34722
18	075	4	016667.	14	a24306	1	e 17361
17	070833						

Averdupois

The Table of Reduction.

Averdupois great weight.

H.	5	17	15179	5	044643	8	a44643
qu	25	16	14286	4	035714	7	a39063
27	24107	15	13393	3	026786	6	a33482
26	23214	14	225	2	017857	5	a27902
25	22321	13	11607	1	a89286	4	a22321
24	21429	12	10714	15	a83705	3	a16741
23	20536	11	098214	14	a78125	2	a11161
22	19643	10	089286	13	a72545	1	e55804
21	1875	9	080357	12	a66964	3	e41853
20	17857	8	071429	11	a61384	2	e27902
19	16964	7	0625	10	a55804	1	e13953
18	16071	6	053571	9	a50223		

Averdupois little weight.

15	9375	6	375	13	050781	5	019531
14	875	5	3125	12	046875	4	015225
13	8125	4	25	11	042969	3	011719
12	75	3	1875	10	039063	2	a78125
11	6875	2	125	9	035156	1	a39063
10	625	1	0625	8	03125	3	a29297
9	5615	15	058594	7	027344	2	a19531
8	5	14	054688	6	023438	1	e97658
7	4375						

Liquid measures.

7	875	4	5	2	25	2	625
6	75	3	375	1	125	1	3125
5	621			3	9375		

The Table of Reduction.

Dry meaſures.

7	835	3	375	2	0625	1	0478125
6	75	2	25	1	03125	3	058594
5	625	1	125	3	023438	2	039063
4	5	3	09375	2	015625	1	019531

Yards and Ells.

3	75	3	1875	1	0625	2	03125
2	5	2	125	3	046875	1	015625
1	25						

Time.

11	91667	30	082193	20	054795	10	027397
10	8333	29	079454	19	052055	9	024657
9	75	28	076714	18	049316	8	021918
8	66667	27	073973	17	046577	7	019178
7	58333	26	071233	16	043837	6	016438
6	5	25	068495	15	041097	5	013699
5	41667	24	065755	14	038357	4	010959
4	33333	23	063016	13	035617	3	082193
3	25	22	060274	12	032877	2	054795
2	16667	21	057536	11	030137	1	027397
1	083333						

Dozens

The Table of Reduction.

Dozens.

10	91667	5	41667	11	076389	5	034722
11	83333	4	33333	10	069444	4	027778
9	75	3	25	9	0625	3	020833
8	66667	2	16667	8	05555	2	013889
7	58333	1	083333	7	048611	1	a69444
	5			6	041667		

PROBL. VIII.

The reſt of a Logarithme (beſides the Characteriſtique) being given, to finde all the figures of the number, unto which it appertaines.

Search the reſt of that Logarithme (beſides the Characteriſtique) in the ſecond Columne of the Table of Logarithmes, where if you finde it exactly, the figures juſt againſt it in the firſt Columne are the figures of the number you looke for : So if 096910 were propounded, the *figures* of the number, unto which it belongs, would be found 125; and 698970 being given, the *figures* of the number, unto which it appertaines, are 500, &c.

But here if you finde not the Logarithme given preciſely, obſerve (as before in the ſecond Columne of the Table) that which being leſſe comes neereſt unto it; then caſting your eye in the ſame line towards the right hand upon the other nine Columnes of Logarithmes, make ſearch in them for the Logarithme propounded, where if you finde it exactly, the three figures in the firſt Columne, and the figure placed at the top of the Columne (where you found the Logarithme given) being annexed together, are the figures of the number, unto which the ſame Logarithme appertaines : So the Logarithm 099335 being propounded, the *figures* of the number, unto which it belongs, are 1257, as appeares by the firſt *example* of the third *Probleme* of this Chapter.

But laſtly, if yet in the other nine Columnes you finde not the exact Logarithme propounded, take (as before) that which being leſſe comes neereſt unto it; then deducting the Logarithme found out of the Logarithme given, annex unto their difference the figure 9; this done, if you divide that difference (ſo enlarged) by the common difference (placed in the laſt Columne of the Table juſt againſt the Logarithme found) the quotient will yeeld you a figure, which being annexed to the foure figures before found, will make up the figures of the number, unto which the Logarithme given doth belong.

So the Logarithme 099404 being propounded, the Logarithme, that comes neereſt it, in the *ſecond* Columne, is 096910, and the Logarithme, that comes neereſt it, in the other *nine* Columnes, is 099335, above which I finde 7 placed at the *top*; wherefore I take 125 (in the firſt Columne) and 7 (in the top of the *Table*) for the *foure firſt* figures of the number required, then to finde the *laſt* figure, I deduct 099335 (the Logarithme laſt found) out of 099404 (the Logarithme propounded) and finde their *difference* to be 69; now therefore if by annexing the figure 9, to 69, I make it 699, and afterwards divide that intire number by 345 (the *common difference* placed in the laſt Columne of the *Table* juſt againſt the Logarithme found) the quotient will give me 2: wherupon at laſt I conclude, that 12572 are the *figures* of the number, unto which the Logarithme 092404 doth appertaine.

And here *obſerve* farther, that if two figures be required to be annexed to the *foure figures* found, you are to annex 99 to the difference of the Logarithmes, and then divide the whole by the *common difference*, as before; for this done, the two figures in the *quotient* are they you deſire. For *example*, 099418 being propounded, the firſt foure figures of his correſpondent number will be 1257, as before, and the *difference* betweene 099335 (the Logarithme of 1257, beſides the *Characteriſtique*) and 099418 is 83; now therefore if by annexing 99 unto 83, I make it 8399, & then divide the whole by 346, (the *common difference*) I find in the *Quotient* 24; I conclude therefore, that 125724 are the *figures* of the number, unto which the Logarithme 099418 doth belong. But this *Rule* holds onely true, when the firſt foure of the figures thus diſcovered amount not to above 4000; for when they exceed that number, the *ſixt* figure cannot be taken preciſely by our *Table*, which profeſſeth onely to preſent the Logarithmes of all *abſolute* numbers under 400000, and no farther.

PROBL. IX.

A perfect Logarithme being given, to finde the correſpondent number unto which it belongs.

I *Call a perfect Logarithme that, which is the Logarithme of a number not leſſe then* 1; *now therefore to finde the proper number of ſuch a Logarithme*

rithme, doe thus: Finde (by the laſt Probleme) the figures of the number, unto which the Logarithme given doth appertaine: this done, having obſerved of how many units the Characteriſtique of the Logarithme propounded doth conſiſt, cut off (towards the left hand) one more of the figures ſo found (as if the Characteriſtique be 0, cut off one of theſe figures, if it be 1 cut off 2, if 2 cut off 3, &c.) for thoſe figures ſo cut off will be the whole part of the number required; and if (beſides) there remaine any figures towards the right hand, they are a Decimall Fraction annexed to the number you look for.

Example. The Logarithme 0,099404 being propounded, I demand the number, unto which it belongs: the figures of the number, whereunto that Logarithme appertaines (by the *Probleme* aforegoing) are 12572, of which figures (becauſe the *Characteriſtique* of the Logarithme given is 0) I cut off one towards the left hand, and then the number required will be found 1|2572, or 1 $\frac{2572}{10000}$: In like manner the correſpondent number of 1,099404 is 12|572, and the proper number of 2,099404 is 125|72, &c.

And here it is to be *obſerved*, that the quantity of the *Characteriſtique* ought wholly to direct, of how many places the number required ought to conſiſt; and therefore ſometimes you are to annex, and ſometimes to cut off certaine cyphers either to or from the figures found in the *Table*, to the end the number required may be anſwerable to the Logarithme propounded: For *example*, if this Log. 0,000000 were given, the number, which 000000 (the reſt of that Logarithme beſides the *Characteriſtique*) doth diſcover in the *Table*, is 100, from which (becauſe here the *Characteriſtique* is 0) I cut off the *two* cyphers, and then the proper number of the Logarithme propounded will be found 1; ſo likewiſe the Logarithme 1,000000 hath for his correſpondent number 10, the Logarithme 0,301030 hath 2, the Logarithme 1,301030 20, &c. Contrariwiſe, the Logarithme 3,000000 being propounded, here the *figures* anſwerable (in the *Table*) to 000000 are 100, as before; but now (becauſe the *Characteriſtique* is 3) in this caſe I am to annex a *cypher* to 100, and then the proper number (unto which the Logarithme given doth belong) is 1000: In like manner 4,000000 is the Logarithme of 100000; 4,301030 the Logarithme of 20000; 5,096910 the Logarithme of 125000; 5,099335 the Logarithme of 125700; 5,099404 the Logarithme of 125720, &c.

PROBL.

PROBL. X.

A defective Logarithme being given, to finde the reſpective number, unto which it appertaines.

A Defective Logarithme is that, which is the Logarithme of a number leſſe then 1; *now to finde the correſpondent number of ſuch a Logarithme, take this direction: Having found the Complement, which the reſt of the Logarithme (beſides the Characteriſtique) doth want of the intire Logarithme of* 10, *make ſearch (by the eighth Probleme of this Chapter) for the figures of the number, unto which that Complement doth belong; this done, if you place before thoſe figures for every unit of the Characteriſtique, of the Logarithme given, a cypher, and at laſt prefix a point or daſh before all; thoſe figures ſo ordered are the number you looke for, which is alwayes in this caſe a Decimall Fraction.*

Example, the Logarithme——0,903090 being given, I demand the number unto which it appertaines; The *complement* of the reſt of that Logarithme (beſides the *Characteriſtique*) to the *intire* Logarithme of 10 is 096910, 096910, and the figures anſwerable to that Logarithme (by the eighth *Probleme* aforegoing) is 125: And now becauſe the *Characteriſtique* of the Logarithme given is 0, I prefix *no cypher* before 125 but onely a point to ſigne it for a *Decimall*; this done, the correſpondent number of the Logarithme propounded will be found .125 or $\frac{125}{1000}$: But if the Logarithme given were —— 1,903090, his reſpective number would be .0125 becauſe the *Characteriſtique* of the Logarithme propounded is 1: So likewiſe the correſpōdent number of 1,301030 is .05, and the number, unto which —— 2,602060 belongs, is .0025, &c.

And here *obſerve*, that the *complement* of a Logarithme may be readily diſcovered, if neglecting the *Characteriſtique* you ſubſcribe under each of the other figures the c of his reſpective *complement* to 9, ſave under the laſt ſignificant figure towards the right hand, under which you are to write his *complement* to 10: for this done, you have the *complement* of the Logarithme propounded. *Example*, —— 0,124939 being given, his *complement* to the intire Logarithme of 10 is 875061; for the *complement* of 1 to 9 is 8, and the complement of 2 to 9 is 7; again, the *complement* of 4 to 9 is 5, and the *complement* of 9 to 9 is 0; laſtly, the *complement* of 3 to 9 is 6, and the *complement* of 9 to 10 is 1: Now all theſe figures (*viz.* 875061) I ſubſcribe one after another under 124939, and then conclude, that the *complement* of ——0,124939, the Logarithme propounded is 875061: So likewiſe is 397940 the *complement* of ——0,602090 and 096910 the *complement* of —— 0,903090, as may appeare by the operations following:

——0,124939	——0,602060	——0,903090
875061	397940	096910

And this *Rule* may likewiſe ſerve for the diſcoverie of the *complement* of a Logarithme, when you are to finde the Logarithme of a *Decimall*, according to the direction given you in the ſixth *Probleme* of this Chapter.

PROBL. XI.

An Arke or Angle of any number of degrees being given, to finde the Logarithme of the right Sine or Tangent of the ſame.

WHen the number of the degrees given exceeds not 45, make ſearch for the *ſame degrees at the top of the Pages of the Table (intituled* Artificiall Sines and Tangents*) and for the Minutes (if there be any) in the firſt Column towards the left hand, ſigned by the letter M; this done, juſt over againſt thoſe Minutes under the Title (Sin.) you ſhall finde the Logarithme of the Sine, and under the Title (Tan.) you ſhall have the Logarithme of the Tangent of the Ark or Angle propounded:* So the Logarithme of the *Sine* of an Arke or Angle of 23 *Degrees* 30 *Minutes* is 9,600696; and the Logarithme of the *Tangent* of the ſame Arke or Angle is 9,638301.

But when the number of the Degrees knowne exceeds 45, looke them at the bottome of the ſame Table, and the Minutes in the firſt Column towards the right hand, ſigned likewiſe by the letter M: for this done, juſt over againſt thoſe Minutes above the Tittle (Sin.) you ſhall diſcover the Logarithme of the Sine, and above the Title (Tan.) you ſhall finde the Logarithme of the Tangent of the Arke or Angle required: So the Logarithme of the *Sine* of *66 Degrees* 30 *Minutes* is 9,662397; and the Logarithme of his *Tangent* is 10,361698, and ſo of the reſt.

Obſerve here, that when the Arke or Angle propounded, exceeds 90 *Degrees*, in ſtead of it you muſt looke his *complement* to 180 *Degrees*, according to that which is practiſed in the uſe of the ordinary *Tables* of *Sines, Tangents* and *Secants.*

PROBL. XII.

The Logarithme of a right Sine, or Tangent, being given, to finde his correſpondent Arke or Angle.

SEarch in the ſame Table of artificiall Sines and Tangents the Logarithme propounded, viz. *in the Columnes ſigned by the title (Sin.) if the Logarithme propounded be the Logarithme of a Sine, or in thoſe ſigned by the title (Tan.) if it be the Logarithme of a Tangent; this done, in the top or bottome of the Columne (where you finde the ſame Logarithme after the title (Sin. or Tan.) you ſhall diſcover the* Degrees, *and juſt againſt the Logarithmes found, in the Columnes marked by the letter M. (either upon the right or left hand reſpectively) you ſhall have the Minutes of the Arke or Angle propounded.* So 23 *Degrees* 30 *Minutes* is the correſpondent Arke to 9,600699, the Logarithme of the *Sine* of the ſame Arke, and likewiſe to 9,638301 the Logarithme of his *Tangent.*

Obſerve, that although the numbers, which you ſhall finde in the *Table* of *Sines* and *Tangents*, are properly the Logarithmes of the *right Sines* and *Tangents* of the Degrees of the *Quadrant*, as is aforeſaid, yet hereafter when we ſhall have any occaſion to mention them, for brevity ſake we wil call them *Sines* & *Tangents* without any other *addition*; So 9,600699 is the *Sine*, and 9,638301 the *Tangent* of 23 *Degrees* 30 *Minutes*.

Againe, in ſtead of the termes *Sine* or *Tangent of the Complement*, we will uſe with Maſter *Gunter* the words *Coſine* and *Cotangent*: So, 9,600699 is the *Coſine*, and 9,638301 the *Cotangent* of 66 *Degrees* 30 *Minutes*. In like manner 9,962397 is the *Coſine*, and 10,361699 the *Cotangent* of 23 *Degrees* 30 *Minutes*, &c.

Laſtly, when you cannot finde preciſely the *Sine* or *Tangent* you looke for, take in ſtead thereof that which comes neereſt unto it.

CHAP. V.

The uſe of the Logarithmeticall Tables in Arithmetique.

HAving in the laſt Chapter ſhewed the firſt uſe of the *Logarithmeticall Tables*, come we now to the *other*, and firſt to their admirable uſe in *Arithmetique*, which you ſhall finde explained in the reſolution of theſe *Problemes* following.

PROBL. I.

One number being given to be multiplied by another number, to finde the Product.

1. *WHen the Logarithmes of the numbers given are both of one kinde (that is,*

is, both perfect, or both defective Logarithmes adde them together; for this done, their ſumme the Logarithme of the Product required, whic Logarithme ſo found, is alwayes of the ſame kind with the Logarithmes of the number given, viz *perfect if they be perfect, defective if they be defective.*

Example, 30 being given to be multiplied by 25, I demand the *Product*; the Logarithme of 30 (by the firſt *Probleme* of the laſt Chapter) is 1,477121, and the Logarithme of 25 is 1,397940, the *ſumme* of theſe Logarithms is 2,875061, which (by the ninth *Probleme* of the ſame Chapter) is found to be the Logarithme of 750, the *Product* required; & likewiſe is .9375 the *Product* of $\frac{3}{4}$ multiplied by .125: for (by the ſixth *Probleme* of the Chapter laſt cited) ——0,124939 is the Logarithme of $\frac{3}{4}$ and ——0,903090 the Logarithme of .125, and the ſumme of theſe Logarithmes is ——1,028029, which (by the tenth *Probleme* of the ſame Chapter) is the Logarithme of .9375 the *Product* ſought for: See the *operations* following,

30.	1,477121	$\frac{3}{4}$	— 0,124939
25.	1,397940	.125	— —0,903090
750.	2,875061	.9375	—1,028019
			971971

But

2. *But when the Logarithmes of the numbers given are of divers kinds, (that is, one perfect, and the other defective) ſubſtract the leſſer out of the greater; This done, the remainder is the Logarithme of the Product required, which Logarithme is in this caſe alwayes of the ſame kinde with the greater Logarithme of the numbers given:* So 25 being given to be multiplied by .125, the *Product* is 3.125, but the ſame 25 being given to be multiplied by .00125, the *Product* is .03125, as appeares by the worke following;

25.	1,397940	25.	1,397940
.125	—0,903090	.00125	—2,903090
3.125	0,494850	.03125	—1,505150
			494850

PROBL. II.

One number being given to be divided by another number, to finde the quotient.

BEfore we proceed to the reſolution of this *Probleme*, the *Propoſition* following ought to be obſerved;

In diviſion by Logarithmes, when the dividend is greater then the Diviſor, the Logarithme of the Quotient is alwayes perfect, & contra.

Becauſe in *diviſion* after the Logarithme of the *Quotient* is diſcovered, there may yet remaine ſome doubt, whether that Logarithme be a *perfect* or a *defective* Logarithme; therfore have I here prefixed this *Propoſition*, to the end that *difficulty* may be removed: For if you conſider the three numbers given in every *diviſion*, you ſhall there finde this *proportion*,

As the Diviſor is to the Dividend,
So is 1 *to the Quotient;*

according to the *obſervation* before made in the ſixth *Corollarie* of the ſecond Chapter. I ſay therefore, when the *Dividend*, which is the ſecond terme, is greater than the *Diviſor*, which is the firſt terme; the *Quotient*, which is the fourth terme, muſt needs be greater than 1, which is the third terme; And therefore in this caſe the Logarithme of the *Quotient* is alwayes *perfect*, that is, the Logarithme of a *whole* or *mixt* number: Contrariwiſe, when the *Dividend* is leſſe than the *Diviſor*, the *Quotient* is leſſe than 1; and therefore in this caſe the Logarithme of the *Quotient* is alwayes a *defective*, *viz*. the Logarithme of a *Fraction*: This being premiſed, let us now proceed to the reſolution of the *Probleme* propounded.

1. *Therefore when the Logarithmes of the numbers given are both of one kinde, ſubſtract the leſſer out of the greater; this done, the reſidue is the Logarithme of the Quotient.*

Example, 94325 being given to be divided by 3947, the *Quotient* is 23.898; For the Logarithme of 94325 (by the fourth *Probleme* of the laſt Chapter) is 4,974627, out of which if I deduct 3,596267, which (by the third *Probleme* of the ſame Chapter) is the Logarithme of 3947, the *remainder* is 1,378360, which (by the ninth *Probleme* of that Chapter) is the Logarithme of 23.898 the *Quotient* required: So likewiſe if .3947 were propounded to be divided by .0573 the *Quotient* would be found 6.8883; For the Logarithme of .0573 (by the ſixth *Probleme* of the laſt Chapter) is —1,241845, out of which if you ſubſtract —0,403733, which (by the ſame *Probleme*) is the Logarithme of .3947, there remaines 0,838112, which (by the ninth *Probleme* of the ſame Chapter) is the Logarithme of 6.8883, as appeares by the work following;

94325.	4,974627	.3947	—0,403733
3947.	3,596267	.0573	—1,241845
23.898	1,378360	6.8883	0,838112

And here in theſe *operations* you may obſerve, that 1,378360 and 0,838112 the Logarithmes found are both of them *perfect*, becauſe the whole number 94325, being the *Dividend* in the firſt *example*, is greater than 3947, his *Diviſor*; and the Fraction .3947, being likewiſe the *Dividend* in the other *example*, is greater than the Fraction .0573, his reſpective *Diviſor*, according to the *Propoſition* before premiſed. But contrariwiſe, if in the firſt *example* 3947 were the *Dividend*, and 94325 the *Diviſor*, and in the other *example* .0573 were the *Dividend* and .3947 the *Diviſor*, in this caſe both the Logarithmes found would be *defective*; becauſe then both the *Dividends* would be leſſe than their reſpective *Diviſors*; This likewiſe will appeare plainly upon the view of the *operations* following,

3947. 3,596267
94325. 4,974627
.041845 —1,378360

.0573 —1,241845
.3947 —0,403733
.14517 —0,838112
161888

For here —— 1,378360 being the *Logarithme found* in the firſt *example*, is (by the tenth *Probleme* of the laſt Chapter) the Logarithme of the *Decimall* 041845, and ——0,838112, being the *Logarithme found* in the other *example*, is (by the ſame *Probleme*) the Logarithme of the *Decimall* .14517.

2. *When the Logarithmes of the numbers given are of divers kindes, adde them together, this done, their ſumme is the Logarithme of the Quotient.*

Example. 3947 being given to be divided by .14517 the *Quotient* is 27188: For the Logarithme of 3947 is 3,596267, and the Logarithme of .14517 is ——0,838112; theſe Logarithmes being added together, their ſumme is 4,434379, which being a *perfect* Logarithme (by the *Propoſition* before premiſed) is the Logarithme of 27188, the *Quotient* required: But if .14517 were propounded to be divided by 3947, the *Quotient* would be found .000036781, *Cauſa qua ſupra.* See the worke,

3947.	3,596267
.14517	—8,838112
27188.	4,434379

.14517	—0,838112
3947.	3,596267
.000036781	—4,434379
	565621

In like manner 6.8883 being given to be divided by .3947, the *Quotient* is 17.452: For the Logarithme of 6.8883 (by the ſeventh *Probleme* of the laſt Chapter) is 0,838112, and the Logarithme of .3947 is—0,403733, theſe Logarithmes being added together, their ſumme is 1,241845, which is the Logarithme of 17.452, the *Quotient* you looke for: But if .3947 were divided by 6.8883, the *Quotient* would be .0573;

6.8883	0,838112
.3947	—0,403733
17.452	1,241845

.3947	—0,403733
6.8883	0,838112
.0573	—1,241845
	758155

I have thought fit to inſiſt the longer upon the explanation of theſe *two firſt Problemes*, becauſe *Multiplication* and *Diviſion* are indeed (as it were) the *two hands*, by which almoſt all the other ſubſequent *operations* of this *Treatiſe* are framed: for having once gained the right way how to performe them, you may (with much eaſe and dexterity) worke any other *operation* whatſoever which may be performed by them: And therefore (I hope) it will not be needfull in the inſuing *Problemes* of this *Treatiſe* to produce *examples* to expreſſe all the varieties, which they likewiſe will admit, (according to the nature of the Logarithmes, by which you are to worke) but onely to confine my ſelfe to ſuch *examples*, which may be performed by *perfect* Logarithmes: referring thoſe, that are deſirous to ſee moe diverſities of *examples* in all *Arithmeticall operations*, to peruſe the *ſecond booke* of my *Arithmetique* (from the beginning of the ſeventh Chapter to the end of the ſame booke) where (as I well hope) they ſhall finde plentifull matter to give them full ſatisfaction in that *point*.

PROBL. III.

Three numbers being given, to finde a fourth in a direct Proportion.

ADde the Logarithme of the ſecond terme to the Logarithme of the third terme, then from their ſumme ſubſtract the Logarithme of the firſt terme; this done, the number that remaines is the Logarithme of the fourth terme demanded.

Example. Let theſe three numbers 12, 6, and 432 be given, to which it is required to finde a *fourth Proportionall:* The Logarithme of 6 is 0,778151, and the Logarithme of 432 is 2,635484; the ſumme of them is 3,413635, from which if I ſubſtract 1,079181 the Logarithme of 12 the firſt *terme*, the number which remaines is 2,334454, which (by the ninth *Probleme* of the laſt Chapter) is the Logarithme of 216, the fourth *terme* required.

As	12.	1,079181
Is to	6.	0,778151
So is	432.	2,635484
		3,413635
To	216.	2,334454

Howbeit here *obſerve*, that the *Rule* of this *Probleme* onely holds true, when the Logarithmes (by which you worke) are all *perfect*, or all *defective* Logarithmes; but when ſome of them are *perfect*, and other ſome *defective*, the operation will vary: This *conſideration* likewiſe ought to be had in all the other ſubſequent *Problemes* of this *Treatiſe*, which hereafter (for *brevity* ſake) I will wholly referre to the *diſcretion* of the *ingenious* Reader.

PROBL. IV.

Having three numbers given, to finde a fourth in an inverſe proportion.

Adde the Logarithmes of the firſt and ſecond termes together, then out of their ſumme ſubſtract the Logarithme of the third terme; this done, that which remaines is the Logarithme of the fourth terme required.

Example. If 375 men in 72 houres can make a *Trench* 20 pearch in length; In how many howres may 133 men make ſuch another of the ſame length?

Men	375.	2,574031
Houres	72.	1,857332
		4,431363
Men	133.	2,123852
Houres	203.	2,307511

So that 133 men in the ſpace of 203 houres may doe as much as 375 men in the ſpace of 72 houres, for the making of ſuch a *Trench*, as aforeſaid, or any other worke whatſoever.

PROBL. V.

Having three numbers given, to finde a fourth in a duplicated Proportion.

DOuble the difference of the Logarithmes, which belong to the two termes, that have the ſame denomination; then if the firſt terme be leſſe then the ſecond, adde that Difference doubled to the Logarithme of the other terme; this done, the ſumme is the Logarithme of the fourth terme required.

So the ſuperficiall *content* of a Circle, whoſe *Diameter* is 14 inches, being 154 ſquare inches; the *content* of another Circle, whoſe *Diameter* is is 28 inches, will be found to be *616* ſquare inches.

Diameter	14	1,146128
Diameter	28	1,447158
Difference		301030
Difference doubled		602060
Content given	154	2,187521
Content required	616	2,789581

But if the firſt terme be greater than the ſecond, ſubſtract the difference doubled from the Logarithme of the other terme.

Example,

Diameter	28	1,447158
Diameter	14	1,146128
Difference		301030
Difference doubled		602060
Content given	616	2,789581
Content required	154	2,187521

PROBL. VI.

Having three numbers given, to finde a fourth in a triplicated Proportion.

Triple the difference of the Logarithmes, which belong to the two termes, that have the ſame denomination; then, if the firſt terme be leſſe than the ſecond, adde that difference ſo tripled to the Logarithme of the other terme, and ſo ſhall you *have the Logarithme of the fourth terme demanded.*

Example. If a *Bullet*, whoſe *Diameter* is 4 inches, weigh 9 pound; another *Bullet*, whoſe *Diameter* is 8 Inches, will weigh 72 pound.

Diameter	4	0,602060
Diameter	8	0,903090
Difference		301030
Difference tripled		903090
Weight given	9	0,954243
Weight required	72	1,857333

But if the firſt terme be greater than the ſecond, ſubſtract the Difference tripled from the Logarithme of the other terme.

Example,

Diameter	8	0,903090
Diameter	4	0,602060
Difference		301030
Difference tripled		902090
Weight given	72	1,857333
Weight required	9	0,954243

PROBL. VII.

Any number being given to finde the ſquare root thereof.

DIvide the Logarithme of the number given by 2; this done, you have the Logarithme of the Root required.

Example,

The number given	12572.	4,099404
The Root required	112.12	2,049702

PROBL. VIII.

To finde the Cube root of any number given.

DIvide the Logarithme of the number given by 3; this done, you have the Logarithme of the Root required.

Example,

The number given	79304.	4,899295
The Root required	42.953	1,633098

PROBL. IX.

Betwixt two numbers given to finde a meane Proportionall.

Adde the Logarithmes of the two numbers given together, then dividing their ſumme by 2, you have the Logarithme of the Meane Proportionall required.

Example,

The numbers given	8	0,903090
	32	1,505150
The ſumme of their Log.		2,408240
The halfe ſumme		1,204120

which is the Logarithme of 16, the *Meane proportionall* demanded.

PROBL. X.

Betwixt two numbers given to finde two meane Proportionals.

HAving ſound the difference betwixt the Logarithmes of the numbers given, and added a third part thereof to the Logarithme of the leaſt number propounded, you have the Logarithme of the leſſer meane Proportionall you look for; to which if you againe adde the ſame third part, you ſhall have the Logarithme of the other meane Proportionall required.

Example,

The numbers given	8	0,903090
	64	1,806180
The difference		903090
The third part		301030
The leſſer m. prop.	16	1,204120
The greater m. prop.	32	1,505150

PROBL.

PROBL. XI.

A ſumme of money being forborne for a certaine time, to finde how much it will be augmented at the expiration of the ſame time, accounting intereſt upon intereſt according to a certaine rate propounded.

SVbſtract the Logarithme of 100 *from the Logarithme of* 100 *and the rate added together; this done, if you multiply their difference with the time propounded, and then adde that Product unto the Logarithme of the Principall, that ſumme is the Logarithme of the Principall and Intereſt required.*

Example. I deſire to know how much 2735,*l.* being forborne 11 yeares, will be increaſed at the expiration of thoſe yeares, accounting Intereſt upon Intereſt after the rate of 8 *l. per centum:* Here if I ſubſtract 2,000000 (the Logarithme of 100) out of 2,033424 (the Logarithme of 108) the remainder is 33424, which being multiplied by a 11 yeares, produceth 367664; wherefore if I now adde the ſame 367664 to 3,436957 the Logarithme of 2735 (the *Principall*, or ſumme of mony forborn) their *Aggregate* is 3,804621, which is the Logarithme of 6377,1, the *Principal increaſed*: So that I conclude, 2735,*l.* being forborne 11 years will augment to 6377,*l*.2,*s*. (for $\frac{1}{10}$ of a pound is 2 *ſhillings*) accounting Intereſt upon Intereſt according to the rate of 8,*l. per centum.*

PROBL. XII.

A ſumme being due at a time to come, to finde what it is worth in readie money.

Here the worke is the ſame with that of the former Probleme, only at laſt in ſtead of adding, you are to ſubſtract the Product out of the Logarithme of the Principall; for that performed, the remainder is the Logarithme of the ſumme you looke for.

Example. What is 2753,*l.* due to be paid 11 yeares hence, worth now in ready money? Here having found the *Product* 367664, as before, I deduct it out of 3,436957, and finde the remainder to be 3,069293, which is the Logarithme of 1173,*l.* the ſumme demanded.

PROBL. XIII.

The Principall, and the Aggregate of the Principall and Intereſt together being given, to finde the rate of the Intereſt.

SVbſtract the Logarithme of the Principall out of the Logarithme of the Aggregate; this done, if you firſt divide the Difference by the time, and then adde that Quotient to the Logarithme of 100, *that ſumme is the Logarithme of* 100, l. *and the rate of the Intereſt added together.*

A delivers to *B* 230, *l.* with *caution* to receive in *lieu* thereof at the expiration of 21 yeares 850, *l.* and in the *Interim* to forbeare the Intereſt; the *Queſtion* is, after what rate of Intereſt *B* is to hold this money: Here firſt 2,361728 (the Logarithme of 230) being deducted out of 2,929419, (the Logar. of 850) their *Difference* is 567691, which if I divide by 21, the *Quotient* is 27033; This *Quotient* if I adde to 2,000000 (the Logarithme of 100) their ſumme or *Aggregate* is 2,027033, which is the Logarithme of 106.42 (*viz.* 106, *l.* and $\frac{42}{100}$ of a pound, which Fraction being reduced (by the *Table* of *Reduction* produced in the ſeventh ſeaventh *Probleme* of the laſt Chapter) is 8, *s.* 4. *d.* 3, *farth.* I conclude therefore that B retaines the 230. l. for the time aforeſaide at the *rate* of *6, l.* 8, *s.* 4, *d,* 3 *fa. per centum*, which is the reſolution of the *Probleme* propounded.

PROBL. XIV.

A yearely rent or annuity being forborne a certaine number of yeares, to finde what the arerages thereof will amount unto, according to any rate propounded.

FIrſt diſcover the Principall, that anſwers to the rent or annuity in queſtion, then finde unto what ſumme that Principall will bee augmented (according to the given rate) at the end of the terme propounded; this done, if you ſubtract the ſame Principall out of that ſumme, the remainder is the ſumme of the Arerages you looke for.

Example, A rent or Annuity of 12, l. *per annum* being forborne 16 yeares, what will the *Arerages* thereof amount unto, they being conceived to increaſe (as they grow due) after the *rate* of 8, l. *per centum*? Here firſt to find the *Principall* that anſwers to 12, l. ſay *thus*,

If 8, l. hath 100, l. for his *Principall*, what ought 12, l. to have for his correſpondent *Principall?*

The anſwere will bee (by the third *Probleme* of this Chapter) 150, l. for the reſpective *Principall* of 12, l. Having thus diſcovered the *Principall* of 12, l. *viz.* 150, l. I find (by the 11. *Probleme* afore going) that the ſame 150, l. being forborne 16 years, will amount (after the rate of 8, l. *per centum*) to 513. 89, that is, 513, *l.* 17, *s.* 9, *d.* now therefore if I deduct 150 (the correſpondent *Principall* of the Annuity given) out of the ſame 513, *l.* 17, *s.* 9, *d.* The remainder, *viz.* 363, *l.* 17, *s.* 9, *d.* is the ſumme of the *Arerages* required,

PROBL. XV.

A yearely rent or Annuity, being propounded, to find what it is worth in ready money.

First finde what the Arerages thereof amount unto at the end of the terme propounded, And then what thoſe arerages are worth in ready money

money, which ſhall likewiſe be the required price or value of the rent or annuity propounded.

Example, what may a man, which is deſirous to lay out his money after the *rate* of 8, *l. per centum*, afford to give for a leaſe of 12. *l. per annum*, that hath yet 16 yeares in being? I finde (by the laſt Probleme) that the Arerages of 12. *l. per annum*, being forborne 16 years, amount then to 363, *l.* 17, *s.* 9, *d.* or 393·89, and I find likewiſe (by the twelveth *Probleme* aforegoing) that the ſame 363, *l.* 17, *s.* 9, *d.* is worth in preſent mony 106·21. *l.* or which is all one) 106. *l.* 4, *s.* 2, *d.* I conclude therefore, that the *value* of the leaſe propounded (at the rate of 8, *l. per centum*) is 106, *l.* 4, *s.* 2. *d.*

Here when the terme of the Annuity beginnes not preſently, but after certaine yeares to come, Find what the arerages forborne for all the time are worth in ready money.

So in the *Example* laſt premiſed, if the annuity, of 16, *yeares* were not to beginne till after the expiration of 5 *yeares*, In this caſe you are to inquire what the *arerages*, (*viz.* 363, l. 17, s. 9, d.) being forborne 21. *yeares*, are worth in ready mony, which you ſhall likewiſe find (by the twelveth *Probleme* beforecited) to bee 72. 288, which being reduced is 72, *l.* 5. *s.* 9, *d.* the *value* of the leaſe required.

PROBL. XVI.

A ſumme of money being propounded, to finde what Annuity, (to continue any number of yeares, and according to any rate given) that ſumme will buy.

Take any annuity at pleaſure; then finde the value of that annuity in ready money: This done, the proportion will be, as followeth,

As the value found is to the annuity taken; So is the ſumme given to the annuity required.

Example, what *annuity* (to continue 16 yeares) will 1205, l. deſerve, ſo that the purchaſer may gaine after the rate of 8. l. *per centum?* Here firſt I take 12. l. *per annum* to continue 16 yeares, and find the *value* thereof in ready mony (by the 15 *Probleme* of this Chapter) to be 106.21, or 106, *l.* 4, *s.* 2, *d.* I ſay therefore,

If 106.21, *l.* give 12, *l. per annum.*

What will 1205, *l.* yeild? *Facit* 171.39, *per annum* which being reduced is 171, *l.* 7, *s.* 9, *d.*

9,*d.* I conclude therefore that 171,*l.*7,*s.* 9,*d.* is the *Annuity* (to endure 16 yeares) which 1205, l. doth deſerve, after the rate of 8. *per centum.*

If any deſire to have theſe *Problemes* of *Intereſt* farther exemplified, let them turne to the two laſt Chapters of the *Appendix* of my *Arithmetique.*

PROBL. XVII.

Any number of Souldiers being given, to order them into a ſquare Battaile of men.

TAke halfe the Logarithme of the number given; this done, that halfe is the Logarithme of the number of men, that ought to be placed in Ranke and File to make a ſquare Battaile of men.

Example, 573. Souldiers are propounded to be thus ordered; Here the Logarithme of 573 is 2,758155, whoſe *halfe* is 1,379077 *viz.* the Logarithme of 23.937; I ſay therefore, that 23 men ought to bee placed in *Ranke*; and as many likewiſe in *File* to make a *ſquare Battaile* of the 573 Souldiers propounded: And here *obſerve*, that the *Decimall* ·937 is altogether neglected, becauſe *Fractions* are not to be conſidered in queſtions, that concerne *Military* orders.

PROBL. XVIII.

Any number of Souldiers being propounded, to order them into a double battaile of men, viz. *which may have twice ſo many in Ranke, as in File.*

TAke halfe the Logarithme of halfe the number given; this done, that halfe Logarithme is the Logarithme of the number of men to be placed in File, and the ſame number doubled will ſhew you how many ought to be placed in Ranke.

Example, 1342 Souldiers being propounded to bee put into that *order*, I demaund how many ought to bee placed in *Ranke*, and how many in *File*: Here *halfe* 1342 is 671, whoſe Logarithme is 2,826723, which being *halfed* is 1,413361, whoſe *correſpondent* number is 26, &c. I conclude therefore, that to order 1342 Souldiers into a *double Battaile*, you are to place 26 men in *File*, and 52 in *Ranke*.

PROBL.

PROBL. XIX.

Any number of Souldiers being given, to order them into a quadruple Battaile, viz. *ſuch as may have foure times ſo many in Ranke, as in File.*

TAke halfe the Logarithme of a fourth part of the number given; this done, that halfe Logarithme is the Logarithme of the number of men to be placed in File, and the ſame number being multiplied by 4 will ſhew you how many men ought to be placed in Ranke.

Example, 2048 Souldiers being offered to be put into this *order*; Here the fourth part of 2048 is 512, whoſe Logarithme is 2,709270 which being halfed is 1,354635, whoſe proper number is 22, &c. I ſay therefore that, to order 2048 Souldiers into a quadruple Battaile, 22 men ought to be placed in *File*, and 88 in *Ranke*. And here note that to diſtinguiſh *this* from other *orders*, the *French* call it, *Bataillon de grand front*, from whence likewiſe we uſually terme it a *Bataile of the great Front*.

PROBL. XX.

Any number of Souldiers being propounded, to order them in Ranke and File according to the Reaſon of any two numbers given.

ADde the Logarithme of the given number of Souldiers, to the Logarithme of the proportionall number given for the Ranke; this done, if out of their Aggregate you deduct the Logarithme of the proportionall number given for the File, alſo the remainder is the Logarithme of the number of men to be placed in Ranke, which Logarithme if you ſubtract out of the Logarithme of the whole number of Souldiers, the remainder is the Logarithme of the number to be placed in File.

Example, 2500 Souldiers are ſo to be ordered, that the number of men placed in *File* may beare ſuch *Proportion* to the number of men placed in *Ranke*, as 5 beare to 12; In this demaund the *Proportion* will be this, *as 5 to 12, ſo 2500 to another number, whoſe ſquare roote is the number of men to bee placed in Ranke:* Wherefore if you adde 1,079181 the

Logarithme of 12, to 3,397940 the Logarithme of 2500, their *Aggregate* is 4,477121, out of which if you ſubtract 0, 698970 the Logarithme of 5, the remainder is 3,778151, whoſe *halfe* being 1, 889075 is the Logarithme of 77, &c. the number of men to bee placed in *Ranke*; and now if out of 3,397940 the Logarithme of 2500 you deduct 1,889075 the Logarithme of the number of men in *Ranke*, the remainder (*viz.* 1,508865) is the Logarithme of 32, &c. the number of men to be placed in *File*.

PROBL. XXI.

Any number of Souldiers being given, together with their diſtances in Ranke and File, to order the ſame Souldiers into a ſquare Battaile of Ground.

ADde the Logarithme of the diſtance in Ranke to the Logarithme of the number of Souldiers; this done, if out of their ſumme you ſubtract the Logarithme of the diſtance in File, halfe of that which remaines is the Logarithme of the number of men to be placed in File, which

Logarithme being deducted out of the Logarithme of the whole number of Souldiers, the remainder is the Logarithme of the number of men to be placed in Ranke.

Example, 2500 men are propounded to be ſo martialled into a *ſquare Battaile* of *ground*, that, their diſtance in *File* being 7 foot and their diſtance in *Ranke* 3 foot, the ground whereupon they ſtand may bee juſt *ſquare:* Here the *Proportion* will be this, *As* 7 *to* 3, *ſo* 2500 *to another number, whoſe ſquare roote is the number of men to be placed in File:* Wherefore if you adde 0,477121 the Logarithme of 3 (the diſtance in *Ranke)* to 3,397940 the Logarithm of 2500, their ſumme is 3,875061, out of which if you ſubtract 0,845098 the Logarithme of 7, the diſtance in *File*, the remainder is 3,029963, whoſe *halfe* being 1,514981 is the Logarithme of 32, &c. the number of men to be placed in *File*; and now if out of 3,397940 the Logarithme of 2500 you ſubtract 1,514981 the Logarithme of the number of men in *File*, that which remaines, *viz.* 1,882959, is the Logarithme of 76, &c. the number of men to bee placed in *Ranke*. And *note*, that this *order* is termed a *ſquare Battaile*, as well as that mentioned in the ſeaventeenth *Probleme* aforegoing, yet is there much difference betwixt them, For in a *ſquare Battaile of men* the number of men in *Ranke* is alwayes equall to the number of men in *File*, though the *ground*, whereon they ſtand, bee not alwayes a juſt *ſquare*; contrariwiſe, in a *ſquare Battaile of ground* it is neceſſary, that the place, on which they ſtand, ſhould bee alwayes a juſt *ſquare*, though the number of men in *Ranke* be not equall to the number of men in *File:* And therefore the *French* (as well as we) call that order *Bataillon quarrê d'hommes*, a ſquare Battaile of men, and this, *Bataillon quarré de Terrain*, a ſquare Battaille of Ground.

PROBL. XXII.

Any number of Souldiers being propounded together with the diſtances in Ranke and File, to order them in Ranke and File according to the reaſon of any two numbers given.

H*Aving added the Logarithmes of the diſtance in File and of the proportionall number given for the Ranke to the Logarithme of the number of Souldiers propounded, ſubtract out of that Aggregate the ſumme of the Logarithmes of the diſtance*

ſtance in Ranke and of the proportionall number given for the File; this done, halfe that which remaines is the Logarithme of men to bee placed in Ranke, which Logarithme being ſubtracted out of the Logarithme of whole number of Souldiers, the remainder is the Logarithme of the number of men to be placed in File.

Example, 12750 are ſo to bee ordered, that the diſtance betwixt man and man may be 7 foot upon the *File* and 3 foot upon the *Ranke*, and that the length of ground for the *Ranke* may beare the ſame *proportion* to the length of ground for the *file*, that 12 beares to 5; Here the Logarithme of 7 (the diſtance in *file*) is 0,845098; the Logarithme of 12 (the *proportionall* number given for the *ranke*) is, 1,079181; and the Logarithme of 12750 (the number of Souldiers propounded) is 4,105510; the *aggregate* of theſe three Logarithmes is *6,029789*: Againe the Logarithme of 3 (the diſtance in *ranke*) is 0,477121, and the Logarithme of 5 (the *proportionall* number given for the *ranke*) is 0,698970; theſe two laſt Logarithmes being added together, their *ſumme* is 1,176091, which being deducted out of the *aggregate* of the three former Logarithmes, the remainder is 4,853698, whoſe *halfe* being 2,426849 is the Logarithm of 267, &c. the number of men to be placed in *ranke*; And now if I ſubtract the ſame Logarithme 2,426849 out of 4,105510 the Logarithme of 12750, the number of Souldiers propounded, the remainder, (*viz.* 1,678661) is the Logarithme of 47, &c. the number of men to be placed in *File*.

CHAP. VI.

The uſe of the Logarithmeticall Tables in the reſolution of right line Triangles.

THus much concerning the uſe of the Logarithmeticall Tables in Arithmetike, now followes their uſe in Geometry, which is chiefly diſcovered in the reſolution of Triangles: And now according to the diviſion of Triangles into right lined, and Sphericall, wee will firſt ſhew their uſe in the reſolution of right line Triangles.

Seld
8° W. 5.
Art.

BODLEI

Λογαριθμοτεχνία,

OR
The Conſtruction, and uſe of the *Logarithmeticall* TABLES;

By the help of which, Multiplication is performed by Addition, Diviſion by Subſtraction, the extraction of the Square root by bipartition, and of the Cube root by tripartition, &c. Finally, the Golden rule, and the reſolution of Triangles, as well right lined, as Sphericall by Addition and Subſtraction.

Firſt publiſhed in the French Tongue by *Edmund Wingate*, an Engliſh Gentleman:

AND

After tranſlated into Engliſh by the ſame Author.

The ſecond Edition, diligently corrected, and much enlarged by the author himſelfe.

Ἐὰν ἦς φιλομαθὴς, ἔσῃ πολυμαθής.

LONDON,
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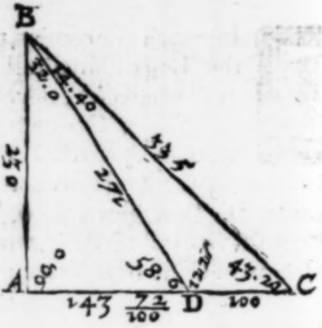
M. DC. XXXV.

PROBL. I.

The three Angles, and one ſide of a right line Triangle being knowne; to finde either of the other ſides.

As the *ſine* of the *Angle* oppoſed to the *ſide given* is to the *parts* of the ſame *ſide*: So is the *ſine* of the *Angle* oppoſed to the *ſide required*, to the *parts* of that *ſide*. And therefore.

Adde the Logarithme of the ſide given to the ſine of the Angle oppoſed to the ſide required; then ſubſtract out of that ſumme the ſine of the Angle oppoſed to the ſaid ſide given: this done, the number, that appertaines to the Logarithme remaining, is the length of the ſide you looke for.



Example, in the Triangle C. B. D. of the Diagram annexed, let the Angle D. bee 122. *degrees*, and the Angle C. 43. *degrees* 20. *min.* and by conſequent the Angle B. (being the complement of the two other Angles to 180. *degrees)* muſt needs be 14. *degrees* 40. *min.* And let the ſide D. C. being 100. paces repreſent the diſtance betwixt the two ſtations D. and C. this ſuppoſed, I demand the diſtance betwixt D. and B. The Logarithme of the ſide D.C. (*viz.* of 100.) by the ſecond Probleme of the fourth Chapter is 2,000000 and the ſine of 43. *degrees* 20. *min.* (the Angle oppoſed to the ſide required) is 9,836477. by the 11. Probleme of the ſame Chapter. The ſumme of theſe is 11,836477. From which if I ſubſtract 9,403455. the ſine of the Angle B. oppoſed to the ſide knowne, the Logarithme which remaines is 2,433022. Unto which (by the 9. Probleme of the ſaid 4. Chapter) anſwers the number 271. which is the length of the ſide D. B. required, *viz.* 271. paces. For

As the ſine of	14.40.	9,403455
Is to the ſide gi.	100.	2,000000
So is the Sin. of	43.20.	9,836477
		11,836477
To the ſide requi.	271.	2,433022

PROBL. II.

Two ſides of a right line Triangle, and an Angle oppoſite to one of them being knowne; to find the Angle oppoſed to the other ſide.

AS the *ſide* oppoſed to the *Angle* given is to the *ſine* of the ſame *Angle:* So is the other *ſide* knowne, to the *ſine* of his oppoſite *Angle.*

So the ſide D. C. being 100. the ſide D. B. 271. and the Angle B. 14. *degrees* 40. *min.* The Angle C. will bee found 43. *degr.* 20. *minutes* For

As the ſide D.C.	100.	2,000000
To the ſin. of B.	14.40.	9,403455
So is the ſide D.B.	271	2,433022
		11,836477
To the ſin. of C.	43.20.	9,836477

PROBL. III.

Having two ſides, and the Angle betweene them, to finde the other two Angles, and the third ſide.

1. IF the Angle included bee a right Angle, the proportion will bee ſuch :

As the greater ſide is to the leſſer : ſo is the Radius to the Tangent of the leſſer Angle.

So in the Triangle A. B. D. of the Diagram premiſed the ſide A. B. being 230. and the ſide D. A. 143 $\frac{72}{100}$

The Angle A.B. D. will be 32. degrees. For,

As the ſide	230	2,361728
Is to the ſide	143 $\frac{72}{100}$	2,157517
So is the Radius		10,000000
		12,157517
To the Tang. of 32. deg.		9,795789

2. But if the Angle included bee oblique; then,

As the ſumme of the ſides given is to the difference of the ſame ſides : ſo is the Tangent of the halfe ſumme of the angles unknowne to the Tangent of the halfe of their difference.

Example, In the Triangle C. B. D. the ſide D. B. being 271. the ſide D. C. 100. and the Angle D. 122. *degrees.* I demand the Angles B. and C. The ſumme of the ſides D. B. and D. C. is 371. the difference of them is 171.

and the Angle D. being 122. degrees, the ſumme of the other Angles unknowne is 58. degrees and therefore the halfe of it is 29. degrees. I ſay then,

As the ſumme	371	2,569374
Is to the differ.	171	2,232996
So is the Tang. of	29.*d.*	9,743752
		11,976748
To the Tang. of	14.20.	9,407374

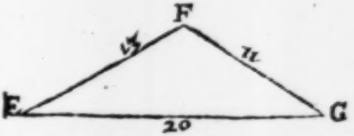
Now theſe 14 *degrees* 20. *min.* being added to 29. d. the halfe ſumme of the Angles unknowne the ſumme will be 43. *degrees* 20. *min.* *viz.* the Angle C. Then ſubſtracting the ſaid 14. *degrees* 20. *min.* from 29. *degrees* you have 14. *degrees* 40. *min.* for the Angle B. Laſtly, knowing the three Angles, and two ſides, the third ſide may bee found by the firſt Probleme of this Chapter.

PROBL. IV.

Having the three ſides of a Triangle to find the ſuperficiall content.

ADde the three ſides together, then from the halfe ſumme ſubſtract each ſide, to the end you may have the difference betwixt that halfe ſumme and each ſide; this done, adde the

Logarithmes of the ſaid halfe ſumme, and of thoſe differences together: And laſt of all dividing the ſumme of thoſe Logarithmes by 2. you have the Logarithme of the content required.



Example, The three ſides of the Triangle E.F.G. being 20. 13. and 11: how much is the ſuperficiall content thereof? The ſumme of the ſides is 44. the halfe ſumme is 22. the differences betwixt each ſide, and that halfe are 2. 9. and 11. which numbers ranke in this order following.

The halfe ſumme	22	1,342423
The differences	2	0,301030
	9	0,954243
	11	1,041393
The ſumme of the Logarith.		3,639089
The content required 66.		1,819544

CHAP. VII.

The uſe of the Logarithmeticall Tables in the reſolution of ſphericall Triangles.

FROM Right Line Triangles came wee to ſphericall, and firſt to Rectangles Triangles. Where obſerve that the ſide oppoſite to the right Angle is called the Baſe, but the other two retaine alwayes the name of ſides.

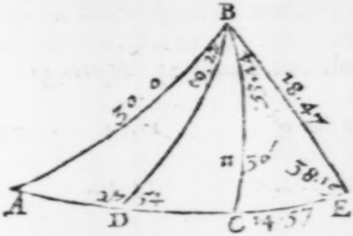
In Rectangle Triangles.

PROBL. I.

The two ſides being given to find the Baſe.

AS the Radius is to the coſine of one of the ſides: ſo is the coſine of the other ſide to the coſine of the Baſe.

Example,



Example, In the Triangle A. B. C. of the Diagram annexed the ſide A. C. being 27. degrees 54. minutes, and the ſide C. B. 11. *degrees* 30. *minutes*, the Baſe B. A. will bee found 30. deg. For,

As the Radius		10,000000
To the Coſ. of	27.54.	9,946337
So is the Coſ. of	11.30.	9,991193
		19,937530
To the Coſ. of 30. degr.		9,937530

PROBL. II.

The two ſides being given to know either of the oblique Angles.

AS the ſine of the ſide next the Angle inquired is to the Radius; ſo is the Tangent of the oppoſite ſide, to the Tangent of the Angle required.

So in the ſame Triangle A.C. being 27. *degr.* 54. *min.* and B.C. 11. *deg.* 30. *min.* the Angle A. will bee found 23. *degrees* 30. *minutes*, For,

As the Sin. of	27.54.	9,670181
To the Radius		10,000000
So is the Tan. of	11.30.	9,308463
		19,308463
To the Tang. of	23.30.	9,638282

PROBL. III.

The Baſe, and one of the oblique Angles being given, to finde the other oblique Angle.

AS the Radius is to the Coſine of the Baſe: ſo is the Tangent of the angle given, to the Cotangent of angle inquired.

Example, the Baſe B. A. being 30 *degrees*, and the Angle A. 23. *degr.* 30. *minutes*, the Angle B. will be 69 *degr.* 22. *min.*

PROBL. IV.

The Baſe, and one of the oblique Angles being given, to finde the ſide next unto the ſame Angle.

As the Radius is to the Coſine of the Angle knowne : ſo is the Tangent of the Baſe, to the Tangent of the ſide required.

So B. A. being 30. *degrees*, and the Angle A. 23. *degrees* 30. *min.* the ſide A. C. will bee found 27. *degrees* 54. *minutes.*

PROBL. V.

The Baſe, and one of the oblique Angles being knowne, to find the ſide oppoſed to the ſame Angle.

As the Radius is to the ſine of the Baſe : ſo is the ſine of the Angle given to the ſine of the ſide inquired.

So B. A. being 30. *degr.* and the Angle A. 23. *degr.* 30. *min.* the ſide C. B. oppoſite to the ſame Angle will be found 11. *degr.* 30. *min.*

PROBL. VI.

One of the ſides and the oblique Angle next unto it being given, to finde the Baſe.

As the Coſine of the Angle given is to the Radius, ſo is the Tangent of the ſide given to *to the Tangent of the Baſe.*

Example, the ſide A. C. being 27. *degrees* 54. *min.* and the Angle A. 23. *degrees* 30. *min.* you ſhall finde the Baſe 30. *degrees.*

PROBL. VII.

One of the ſides, and the oblique Angle next unto it being knowne, to find the other ſide.

AS the Radius is to the ſine of the ſide given; ſo is the Tangent of the Angle given to the Tangent of the ſide required.

So A. C. being 27. *degrees* 54. *min.* and the Angle A. 23 *degrees* 30. *minutes*, the ſide C.B. will bee found 11. *degr.* 30. *minutes.*

PROBL. VIII.

One of the ſides, and the oblique Angle next unto it being knowne; to finde the other oblique Angle.

AS the Radius to the Coſine of the ſide given, ſo is the ſine of the Angle given to the Coſine of the angle required.

So A. C. being 27. *degrees* 54. *minutes*, and the Angle A 23. *degrees* 30. *min.* you ſhall find the Angle B. 69. *degr.* 21. *min.*

PROBL. IX.

One of the ſides, and the Angle oppoſed unto it being knowne; to finde the Baſe.

AS the ſine of the Angle given to the ſine of the ſide given; ſo is the Radius to the ſine of the Baſe.

So the ſide B. C. being 11. *degrees* 30. *min.* and the Angle A. 23. *degrees* 30. *min.* the Baſe will be found 30. *degr.*

PROBL. X.

One of the ſides, and the Angle oppoſed unto it being given; to finde the other ſide.

AS the Tangent of the angle given is to the Tangent of the ſide; ſo is the Radius to the ſine of the ſide required.

So B. C. being 11. 30. and the Angle A. 23. 30. the ſide A. C. will be 27. 54.

PROBL.

PROBL. XI.

One of the ſides, and the Angle oppoſed unto it being given; to find the other oblique Angle.

AS the Coſine of the ſide given is to the Coſine of the Angle given; ſo is the Radius to the ſine of the angle required.

So B.C. being 11.30. and the Angle A.23.30. the Angle B. will be found *69. 22.*

PROBL. XII.

One of the ſides, and the Baſe being knowne; to find the oblique Angle adjacent unto the ſame ſide.

AS the Tangent of the Baſe is to the Tangent of the ſide given; ſo is the Radius to the Coſine of the angle required.

So the ſide A. C. being 27. *degrees* 54. *min.* and the baſe B.A.30. *degrees* 0. *min.* the Angle A. will be found 23. *degr.* 30. *min.*

PROBL. XIII.

One of the ſides, and the Baſe being knowne; to find the Angle oppoſed unto the ſame ſide.

AS the ſine of the Baſe is to the Radius; ſo is the ſine of the ſide given to the ſine of the angle required.

So A. C. being 27. *degrees* 54. *minutes*, and B. A. 30. *degrees* 0. *min.* the Angle B. will be found 69. *degr.* 22. *min.*

PROBL. XIV.

One of the ſides, and the Baſe being given; to find the other ſide.

AS the Coſine of the ſide given is to the Radius; ſo is the Coſine of the Baſe to the Coſine of the ſide required.

So A. C. being 27. deg. 54. min. and B. A. 30. degr. 0. min. the ſide B. C. will be found 11. deg. 30. min.

PROBL. XV.

The two oblique Angles being given, to find the Baſe.

AS the Tangent of one of the angles is to the Cotangent of the other angle; ſo is the Radius to the Coſine of the Baſe.

So the Angle A. being 23. degr. 30. min. and the Angle B. 69. degr. 22. min. the Baſe B. A. will be found 30. degr. 0. min.

PROBL. XVI.

The two oblique Angles being given, to find either of the ſides.

AS the ſine of one of the angles is to the Coſine of the other angle; ſo is the Radius to the Coſine of the ſide oppoſite to the angle whoſe Coſine was taken.

So the Angle A. being 23. *degrees* 30. *min.* and the Angle B. 69. *degr.* 22. *min.* the ſide A. C. will be found 27. *degr.* 54. *min.*

In all Sphericall Triangles.

PROBL. XVII.

Two Angles, and a ſide oppoſed to one of them being given to find the ſide oppoſed to the other.

AS the ſine of the angle oppoſed to the ſide knowne is to the ſine of the ſame ſide; ſo is the ſine of the angle, oppoſed to the ſide required to the ſine of that ſide.

So in the Triangle A. B. E. of the laſt Diagram the Angle B. being 38. *degrees* 15. *min.* the ſide B. A. 30. *degrees* 0. *min.* and the Angle A. 23. *degrees* 30. *min.* the ſide B. E. will be found 18.47.

PROBL. XVIII.

Two ſides, and an Angle oppoſed to one of them being knowne, to find the Angle oppoſed to the other.

AS the ſine of the ſide oppoſed to the angle given, is to the ſine of the ſame angle; ſo is the ſine of the ſide oppoſed to the angle inquired;

ſo

to the ſine of that angle.

Example, In the ſaid Triangle A. B.E. the ſaid B. A. being 30. *degrees* 0 *min.* the Angle E. 38. *degrees* 15. *min.* and the ſide B. E. 18. *deg.* 47. *min.* the Angle A. will be found 23.30.

PROBL. XIX.

Two ſides, and the Angle included betwixt them being given, to finde the other ſide.

TO reſolve this *Probleme*, it is neceſſary to let fall a *perpendicular* from one of the angles *unknowne* upon one of the ſides *given*; and by this meanes to change the Triangle propounded into two *Rectangle* Triangles.

For *example*, the ſaid Triangle A.B.E. being propounded, of which the ſides A. B. and A. E. as alſo the angle A. are knowne, and the ſide B. E. is required. Letting fall from the angle B. the *perpendicular* B. C. upon the ſide A. E. by the 5. *Probleme* of this Chapter (for as the Radius is to B. A. ſo is the angle A. to the *perpendicular* B. C.) the Triangle A. B. E. becomes two Rectangle Triangles, *viz.* A.C. B. and B. C.E. Now the operation being thus prepared, in the Triangle A. C.B.

the ſide B. A. and the angle A. being knowne, the ſide A. C. may bee found by the fourth *Probleme* of this Chapter, which being ſubſtracted from the ſide A. E. the remainder will be C. E. *viz.* one of the ſides of the Rectangle B. C. E. and by conſequent in the ſaid Rectangle B. C. E. the ſides B. C. and C. E. being knowne, the baſe B. E. (*viz.* the ſide required) may bee found by the firſt *Probleme* of this Chapter.

Sometimes the *perpendicular* will fall out of the Triangle propounded; ſo if the Triangle A. B. D. were propounded, the *perpendicular* would fall upon the ſide A. D. being prolonged as farre as the point C. and in this caſe the two Rectangle Triangles are A. C. B. and D. C. B. and therefore the ſide B. D. required may bee likewiſe found by the *Problemes* of Rectangle Triangles.

PROBL. XX.

Two ſides, and the Angle included betwixt them being given, to finde either of the other Angles.

HEre you muſt let fall the *perpendicular* from one of the angles unknowne upon on the ſide adjacent to the angle required, and then reſolve it by the Problemes of Rectangle Triangles, as before.

PROBL. XXI.

Two ſides being given, and one of the angles adjacent to the ſide unknowne, to finde the ſame ſide.

THis Probleme, may be reſolved alſo by letting fall a *perpendicular* from the angle included betwixt the two ſides knowne upon the ſide unknowne.

PROBL. XXII.

Two ſides being given, and one of the angles adjacent to the other ſide, to finde the angle included betwixt the ſides given.

FOr the reſolution of this Probleme let the *perpendicular* fall from the angle inquired upon his oppoſite ſide.

PROBL.

PROBL. XXIIII.

Two Angles being given, and the ſide adjacent unto them, to finde either of the other ſides.

IN this caſe you muſt let fall the *perpendicular* from one of the angles knowne upon one of the ſides unknowne.

PROBL. XXIV.

Two Angles being given, and the ſide adjacent unto them, to finde the other Angle.

THis Probleme is performed by letting fall the *perpendicular* from either of the angles knowne upon his ſubtending ſide.

PROBL. XXV.

Two Angles being given, and one of the ſides adjacent unto the Angle unknowne, to finde the ſide adjacent unto the angles given.

THis Probleme is reſolved by letting fall the *perpendicular* from the angle unknowne upon the ſide required.

PROBL. XXVI.

Two Angles being given, and one of the ſides adjacent unto the angle unknowne, to finde the ſame angle.

TO reſolve this Probleme you muſt let fall the *perpendicular* from the angle unknowne upon his oppoſite ſide.

PROBL. XXVII.

The three ſides being knowne, to finde any of the Angles.

ADde the three ſides together, then from the halfe ſumme ſubſtract the ſide oppoſite to the angle required; this done, the proportion will be as followeth.

1. *As the Radius is to the ſine of one of the ſides, which comprehend the Angle required; ſo is the ſine of the other ſide that comprehends the ſame angle, to a fourth ſine.*

2. *As that fourth ſine is to the ſine of the halfe ſumme of the ſides; ſo is the ſine of the difference betwixt*

betwixt the halfe ſumme and the ſide oppoſed to the angle required, to a ſeventh ſine.

Now if you adde to this ſeventh ſine the Radius, halfe the ſumme which accrues upon that addition will bee the ſine of an Arke, whoſe complement being doubled, will give you the Angle you looke for.

Example, In the ſaid Triangle A. B. E. the ſide A.B. being 30.0. the ſide B.F. 18.47. and the ſide A.E. 42.51. I demād the angle B. The ſumm of the ſides is 91.38. halfe of this ſumm is 45. 49. and the ſide A.E. 42.51. being ſubſtracted from that half ſumm there remaines 2.58. for the difference betwixt the ſaid halfe ſumme, and the ſaid A. E. I ſay then,

I.		
As the Radius		10,000000
To the Sine of	30. 0.	9,698970
So the Sine of	18.47.	9,507843
		19,206813
To the fourth Sine.		9,206813
II.		
As the fourth Sine.		9,206813
To the Sine of	45.49.	9,855588
So the Sine of	2.58	8,713952
		18,569540
To the ſeaventh ſine		9,362727
The Radius		10,000000
The ſumme		19,362727
The halfe ſumme		9,681363

which being ſearcht in the Table of Sines and Tangents will bee found the ſine of 28. *degr.* 42. *min.* whoſe complement is 61. *degr.* 18. *min.* which being doubled is 122. *degr.* 36. *m.* *viz.* the meaſure of the angle A.B.E. required.

PROBL. XXVIII.

The three Angles being knowne, to find any of the ſides.

IF in ſtead of the greateſt Angle, you take his complement to 180. degrees, the Angles change themſelves into ſides, and the ſides into angles, and by conſequent the operation will bee the ſame with that of the laſt Probleme.

AN APPENDIX,

CONTEINING

Certaine Rules and Analogies, by helpe whereof the Logarithmeticall Tables may be yet made more uſefull.

I. In Geometry.

Probl. 1. *The Diameter of a Circle being knowne, to finde the Circumference.*

As 1. is to the Diameter; ſo is 3.1416. (whoſe *Logarithme* is 0,497151) to the Circumference.

Probl. 2. *To finde the ſuperficiall Content.*

As 1. is to the ſquare of the Diameter; ſo is .785398 (whoſe *Logar.* is—0,104910) to the Content.

Probl. 3. *To find the ſide of the ſquare, which may be inſcribed within the ſame Circle.*

As 1. is to the Diameter; ſo is .707107. (whoſe *Logar.* is——0,150515) to the ſide of the ſquare to bee inſcribed.

Probl. 4. *Having the Circumference of a Circle to find the Diameter.*

As 1. is to the Circumference; ſo is .318308 (whoſe *Logar.* is—0,497153) to the Diameter.

Probl. 5. *To finde the ſuperficiall Content.*

As 1. to the ſquare of the Circumference; ſo is .079578 (whoſe *Logar.* is— 1,099207) to the Content.

Probl. 6. *To finde the ſide of the ſquare which may be inſcribed within it.*

As 1. to the Circumference; ſo is .225078 (whoſe *Logar.* is——0,647667) to the ſide of the ſquare inſcribed.

Probl. 7. *Having the Content of a Circle to finde the Diameter.*

As 1. to the Content; ſo is 1.27324 (whoſe *Logar.* is 0,104910) to the ſquare of the Diameter.

Probl. 8. *To finde the Circumference.*

As 1. to the Content; ſo is 12.5664 (whoſe *Logar.* is 1,099211) to the ſquare of the Circumference.

Prob.9.*To find the ſide of a ſquare equall unto it.*

Extract the ſquare root of the Content given; for that root is the ſide of the ſquare required.

Probl. 10. *Having the Axis of a Sphere, to finde the ſuperficiall Content.*

As 1. is to the ſquare of the Axis; ſo is 3.1416 (whoſe *Logar.* is 0,497151) to the ſuperficiall Content.

Probl. 11. *To finde the ſolid Content.*

As 1. to the Cube of the Axis; ſo is .52381 (whoſe *Logar.* is——0,280826) to the ſolid Content.

Probl. 12. *In a Wine or Beere veſſell, the Diameters at the bongue and head being knowne, to find the æquated Diameter.*

Multiply the difference of the two Diameters given by 3; this done, if removing that product a place towards the right hand, you deduct it out of the Diameter at the bongue the remainder will be the æquated Diameter required.

So the Diameter at the bongue being 25. 50 inches at the Diameter at the head 20. 75, the æquated Diameter will be found 24 075, as appeares by the operation hereunto annexed.

```
25.500
20.75
------
 4 75
 1.425
------
24.075
```

Probl. 13. *The æquated Diameter and length of a wine or Beere veſſell being knowne, to finde the Content thereof in wine meaſure.*

Unto the Logarithme of the length and the Logarithme of the æquated Diameter (twice repeated) adde the Logarithme 7, 531473; this done, if you cancell the firſt figure of their ſumme (towards the left hand) the remainder towards the right hand, is the Logarithme of the Content in Wine-gallons.

Probl. 14. *To finde the Content in Ale-meaſure.*

Unto the Logarithme of the length and the Logarithme of the æquated Diameter (twice repeated) adde the Logarithme 7, 460122; this done, if you cancell the firſt figure of their ſumme (towards the left hand) the remainder towards the right hand, is the Logarithme of the Content in Ale-gallons.

If any deſire to ſee theſe Problemes exemplified, let them turne to the firſt Chapter of my uſe of Logarithmes in Geometrie.

II. In Aſtronomy.

Probl. 15. *The ſunnes greateſt declination together with his diſtance from the next equinoctiall point*

point being knowne, to find his preſent declination.

As the Radius to the Sine of the ſunnes diſtance from the next Equinoctiall point; ſo is the Sine of the ſunnes greateſt declination to the ſine of the Declination required.

Probl. 16. *To finde his right Aſcenſion.*

As the Radius to the Tangent of the diſtance; ſo is the coſine of the greateſt declination to the Tangent of the right Aſcenſion.

Probl. 17. *The ſunnes greateſt declination together with his preſent declination being known, to finde his right Aſcenſion.*

As the Tangent of the greateſt declination is to the Radius; ſo is the Tangent of the declination to the Sine of the right Aſcenon.

Probl. 18. *The Elevation of the Pole together with the ſuns declination being known, to find how long the ſunne riſeth or ſetteth before or after the houre of ſix.*

As the Cotangent of the Elevation is to the Radius; ſo is the Tangent of the ſunnes declination to the Sine of the Aſcenſionall difference betweene the hower of ſix and the ſunnes riſing or ſetting.

Probl. 19. *To find the ſunnes Amplitude.*

As the coſine of the Elevation is to the Sine of the Declination; ſo is the Radius to the Sine of the Amplitude.

Probl. 20. *The Elevation of the Pole, the ſunnes greateſt Declination and his diſtance from the next Equinoctiall point being knowne, to finde the Amplitude.*

As the coſine of the Elevation is to the ſine of Suns diſtance; ſo is the ſine of the Suns greateſt declination to the Amplitude required.

Probl. 21. *When the ſunne is in the Equinoctiall, by knowing the Elevation of the Pole, to finde the ſunnes Height at any time aſſigned.*

As the Radius to the Coſine of the Elevation; ſo is the ſine of the Suns diſtance from ſix a clock to the ſine of the Height required.

Probl. 22. *The Elevation of the Pole and declination of the ſunne being knowne, to finde the Suns height at the hower of ſix.*

As the Radius to the ſine of the Latitude; ſo is the ſine of the declination to the ſine of the height required.

Probl. 23. *To find the ſunnes height at any time aſſigned.*

1. As the Radius is, to the Cotangent of the Elevation; ſo is the ſine of the Suns diſtance from ſix, to the Tangent of an Arke, which being ſubtracted out of the Suns diſtance

ſtance from the Pole, I ſay againe.

2. As the Coſine of the Arke found is to the Coſine of the reſidue of the Suns diſtance from the Pole; ſo is the ſine of the Elevation to the ſine of the height required.

Prob. 24. *To finde the time when the ſunne will be due Eaſt and weſt.*

As the Tangent of the Elevation is, to the Radius; ſo is the Tangent of the declination to the Coſine of the hower from the Meridian.

Probl. 25. *To finde the ſunnes height when he cometh to be due Eaſt and weſt.*

As the ſine of the Elevation to the Radius; ſo is the ſine of the Declination to the height required,

Probl. 26. *To finde the Suns Azimuth at the hower of ſix.*

As the Coſine of the Elevation is to the Cotangent of the Declination; ſo is the Radius to the Tangent of the Azimuth from the North part of the Meridian.

Prob. 27. *The complement of Elevation of the Suns diſtance from the Pole, and the complement of the ſunnes height being knowne, to finde the Azimuth.*

Having added the three given termes together find the difference betwixt their halfe ſumme, and the Suns diſtance from the Pole; this done, the proportions will bee as followeth.

1. As the Radius to the Coſine of the Elevation; ſo is the Coſine of the height to a fourth ſine:

2. As that fourth ſine is to the ſine of the halfe ſumme; ſo is the ſine of the difference to a ſeaventh ſine, unto which if you adde the Radius, halfe that ſumme will bee the ſine of an Arke, whoſe complement being doubled is the Azimuth you lookefor.

Probl. 28. *To finde the Hower of the day.*

Having added the three given termes together, as before, find the difference betwixt their halfe ſumme and the complement of the Suns height; this done, the proportions will be theſe.

1. As the Radius to the Coſine of the Elevation; ſo is the ſine of the Suns diſtance from the Pole to a fourth ſine.

2. As that fourth ſine is to the ſine of the halfe ſumme; ſo is the ſine of the difference to a ſeaventh ſine, unto which if you adde the Radius, halfe that ſumme will be the ſine of an Arke, whoſe complement being doubled and converted into time will produce the houre required.

III. In

III. In Dyalling.

Probl. 29. *To make a direct Polar Dyall.*

Having aſſigned a line drawn in the middle of the Plane for the Meridian, and another line drawne parallel unto it for ſome other hower which may bee deſcribed upon the Plane, I ſay;

1. As the Tangent of that hower is, to the Radius; ſo is the diſtance of that hower line from the Meridian, to the height of the ſtile.

2. As the Radius is, to the height of the ſtile; ſo is the Tangent of any other hower, to the diſtance of the ſame hower from the ſubſtile.

Probl. 30. *A Meridian Dyall.*

Having drawne a line repreſenting part of the Axis of the world towards a proper ſide of the plane (according to his ſituation either Eaſtward or Weſtward) and aſſigned that line for the hower of ſixe, the proportions will fall out to be as in the former probleme; for,

1. As the Tangent of any howers diſtance from ſix is to the Radius; ſo is the diſtance of the hower upon the Plane from the hower line of ſix, to the height of the ſtile;

2. As the Radius is to the height of the ſtile,

ſtile; ſo is the Tangent of any other howers diſtance from ſix, to the diſtance of the ſame hower from the ſubſtile.

Probl. 31. *An Horizontall Dyall.*

As the Radius to the Tangent of the hower given; ſo is the ſine of the Elevation, to the Tangent of the hower line from the Meridian.

Probl. 32. *A verticall Dyall.*

As the Radius to the tangent of the hower; ſo is the Coſine of the Elevation to the Tangent of the hower line from the Meridian.

Probl. 33. *A verticall inclining Dyall.*

Having found out the Elevation of the Pole above the Plane according to its inclination, the proportion will be this;

As the Radius to the Tangent of the hower; ſo is the ſine of the Elevation above the Plane, to the Tangent of the hower line from the Meridian.

Probl. 34. *A verticall declining Dyall.*

1. As the Radius to the Cotangent of the Elevation; ſo is the ſine of the declination, to the Tangent of the ſubſtiles diſtance from the Meridian of the place.

2. As the Radius to the Coſine of the declination; ſo is the Coſine of the Elevation,

to the ſine of the ſtiles height above the ſubſtile.

3. As the ſine of the Elevation is to the Radius; ſo is the Tangent of the declination to the Tangent of the inclination of the Meridian of the Plane to the Meridian of the place.

4. As the Radius to the ſine of the ſtiles height above the ſubſtile, ſo is the Tangent of the Angle at the Pole comprehended betweene the hower given and the Meridian of the Plane, to the Tangent of the hower lines diſtance from the ſubſtile.

Probl. 35. *A Meridian inclining Dyall.*

1. As the Radius is to the Tangent of the Elevation; ſo is the ſine of the Inclination, to the Tangent of the ſubſtiles diſtance from the meridian.

2. As the Radius is to the ſine of the Elevation; ſo is the Coſine of the Inclination, to the ſine of the ſtiles height above the ſubſtile.

3. As the Coſine of the Elevation is to the Radius; ſo is the Tangent of the Inclination, to the Tangent of the Inclination of meridians.

4. As the Radius is to the ſine of the ſtiles height above the ſubſtile; ſo is the Tangent

of the Angle at the Pole, to the Tangent of the hower lines diſtance from the ſubſtile.

Probl. 36. *A Polar declining Dyall.*

1. As the Radius to the ſine of the declination, ſo is the Coſine of the Elevation, to the Coſine of the Arke comprehended betweene the Horizon and the ſubſtile.

2. As the Radius, to the Tangent of the declination; ſo is the ſine of the Elevation, to the Tangent of the Inclination of meridians, which being converted into time ſheweth how many howers the ſubſtile ought to be placed from the hower line of 12.

3. As the Radius is to the Tangent of the howers diſtance from the ſubſtile; ſo are the parts of the height of the ſtile, to the diſtance of the ſubſtile from the hower-line required, meaſured by a ſcale of like parts.

Probl. 37. *A declining inclining Dyall.*

1. As the Radius, to the Tangent of Inclination to the Horizon, ſo is the Coſine of declination, to the Tangent of the Arke of the meridian of the place intercepted betweene the Horizon and the Plane, which being compared with the Elevation of the Pole, the diſtance of the Pole from the Plane may be thereby readily diſcovered.

2. As the Radius is to the ſine of declination

from the verticall; ſo is the ſine of inclination to the Horizon, to the Coſine of the Inclination to the meridian.

3. As the Radius is to the Coſine of Inclination to the Horizon; ſo is the cotangent of declination, to the Tangent of the Arke of the Plane intercepted betweene the Horizon and the meridian of the place.

4. As the Radius is to the ſine of the inclination to the meridian; ſo is the Tangent of the Poles diſtance from the Plane, to the Tangent of the ſubſtiles diſtance from the meridian.

5. As the Radius is to the ſine of the Poles diſtance from the Plane; ſo is the ſine of the inclination to the meridian, to the ſine of the ſtiles height above the ſubſtile.

6. As the Coſine of the Poles diſtance from the Plane is to the Radius; ſo is the Cotangent of the Inclination to the meridian, to the Tangent of the Inclination of meridians.

7. As the Radius is to the ſine of the ſtiles height above the ſubſtile; ſo is the Tangent of the Angle at the Pole, to the Tangent of the hower-lines diſtance from the ſubſtile.

IV. In

IV. In Geographie.

Probl.38. *Two places being propounded, which differ only in latitude, to finde their diſtance.*

1. When the two places are ſituate under the ſame meridian and upon the ſide of the Equinoctiall; *Subtract the leſſer latitude out of the greater; that done, the remainder is the diſtance required.*

2. When one of the places propounded are ſcituate upon this ſide of the Equinoctiall, and the other upon that, and yet both under the ſame meridian, as before; *Adde the two latitudes together; this done, their ſumme is the diſtance required.*

Probl. 39. *Two places which differ only in longitude being propounded, to know their diſtance.*

1. When the places are both of them ſcituate under the Equinoctiall; *Subtract the leſſer longitude out of the greater; this done, the remainder is the diſtance required.*

2. When the places are ſcituate under ſome parallel betwixt the Equinoctiall and one of the Poles; Then, *As the Radius is to the Coſine of the common latitude given; ſo is the ſine of halfe the difference of longitude, to the ſine of halfe the diſtance.*

Probl. 40. *Two places being given, that differ both in longitude and latitude, to finde their diſtance.*

1. When one of the places is ſcituate under the Equinoctiall; and the other towards one of the Poles; Then, *As the Radius is to the Coſine of the difference of longitude; ſo is the Coſine of the latitude given, to the Coſine of the diſtance required.*

2. When both places are without the Equinoctiall, and towards one of the Poles; Then;

1. *As the Radius is to the Coſine of the difference of longitude; ſo is the Cotangent of the leſſer latitude, to the Tangent of another Arke*, which having ſubtracted out of the complement of the greater latitude, retaine the remaining Arke thereof, and ſay;

2. *As the Coſine of the Arke found is to the Coſine of the arke remaining; ſo is the ſine of the leſſer latitude, to the Coſine of the diſtance required.*

3. When both places ate without the Equinoctiall, and one of them ſcituate towards the North pole, and the other towards the ſouth.

1. *As the Radius, is to the Coſine of the difference of longitude, ſo is the Cotangent of one of the latitudes*

latitudes, to the Tangent of another arke, which being ſubtracted out of the other latitude and 90 *degr.* added together; ſay againe,

2. *As the Coſine of the Arke found, to the Coſine of the Arke remaining; ſo is the ſine of the latitude firſt taken, to the Coſine of the diſtance required.*

V. In Navigation.

Probl.41: *The latitudes of two places being knowne, to find the Meridionall difference of the ſame latitude.*

1. When one of the places is ſcituate under the Equinoctiall, and the other without, *the degrees and minutes (in the table following) Anſwering to the latitude of that other place are the Meridionall difference of thoſe latitudes.*

2. When both the places have Northerly or Southerly latitude *If you ſubtract the degrees and minutes (in the table) anſwering to the leſſer latitude out of thoſe anſwering to the greater latitude, the remainder is the meridionall difference required.*

3. When one of the places hath Southerly and the other Northerly latitude; *The ſumme of the numbers anſwering to thoſe latitudes, is the Meridionall difference deſired.*

M. Gr.

M	G.	Par	M	G	Par	M	G.	Par
0	0	0	3	3	001	6	6	011
		100		3	101		6	111
		200		3	201		6	212
		300		3	301		6	312
		400		3	402		6	413
		500		3	502		6	514
		600		3	602		6	614
		700		3	702		6	715
		800		3	803		6	816
		900		3	903		6	915
1	1	000	4	4	003	7	7	017
	1	100		4	103		7	118
	1	200		4	204		7	219
	1	300		4	304		7	319
	1	400		4	404		7	420
	1	500		4	504		7	521
	1	600		4	605		7	622
	1	700		4	705		7	723
	1	900		4	805		7	824
	1	800		4	906		7	925
2	2	000	5	5	006	8	8	026
	2	100		5	106		8	127
	2	200		5	207		8	228
	2	300		5	307		8	329
	2	400		5	408		8	430
	2	500		5	508		8	531
	2	601		5	609		8	632
	2	701		5	709		8	733
	2	801		5	810		8	834
	2	901		5	910		8	936
3	2	001	6	6	011	9	9	037

M	G.	*Par*	*M*	G.	*Par*	*M*	G.	*Par*
9	9	027	12	12	088	15	15	174
	9	138		12	190		15	277
	9	239		12	293		15	381
	9	341		12	395		15	485
	9	442		12	497		15	588
	9	543		12	600		15	692
	9	645		12	702		15	796
	9	746		12	805		15	900
	9	848		12	907		16	004
	9	949		13	010		16	107
10	10	051	13	13	112	16	16	211
	10	152		13	215		16	316
	10	254		13	318		16	420
	10	355		13	421		16	524
	10	457		13	523		16	628
	10	559		13	626		16	732
	10	661		13	729		16	836
	10	762		13	832		16	941
	10	864		13	935		17	045
	10	966		14	038		17	150
11	11	068	14	14	141	17	17	255
	11	170		14	244		17	359
	11	272		14	347		17	464
	11	374		14	450		17	568
	11	476		14	553		17	673
	11	578		14	656		17	778
	11	680		14	760		17	883
	11	782		14	863		17	988
	11	884		14	967		18	093
	11	986		15	070		18	198
12	12	088	15	15	174	18	18	303

M	G.	Par	M	G	Par	M	G.	Par
18	18	303	21	21	485	24	24	734
	18	408		21	593		24	844
	18	513		21	701		24	953
	18	619		21	808		25	063
	18	724		21	915		25	173
	18	830		21	023		25	282
	18	939		22	130		25	392
	19	041		22	238		25	502
	19	146		22	345		25	613
	19	251		22	453		25	723
19	19	356	22	22	561	25	25	833
	19	463		22	669		25	943
	19	569		22	777		26	054
	19	675		22	885		26	164
	19	781		22	993		26	278
	19	887		23	101		26	386
	19	993		23	210		26	497
	20	100		23	318		26	608
	20	206		23	427		26	719
	20	312		23	535		26	830
20	20	419	23	23	643	26	26	941
	20	525		23	752		27	052
	20	632		23	861		27	164
	20	738		23	970		27	275
	20	845		24	079		27	387
	20	952		24	188		27	499
	21	059		24	297		27	610
	21	165		24	406		27	722
	21	272		24	515		27	834
	21	379		24	624		27	946
21	21	486	24	24	734	27	28	058

M	G.	Par	M	G.	Par	M	G.	Par
27	28	058	30	31	473	33	34	992
	28	171		31	588		35	111
	28	283		31	704		35	231
	28	395		31	820		35	350
	28	508		31	936		35	470
	28	621		32	052		35	590
	28	734		32	168		35	710
	28	847		32	284		35	830
	28	959		32	400		35	950
	29	072		32	517		36	071
28	29	186	31	32	633	34	36	191
	29	299		32	750		36	312
	29	413		32	867		36	433
	29	526		32	984		36	554
	29	640		33	101		36	675
	29	753		33	218		36	796
	29	867		33	336		36	917
	29	981		33	453		37	039
	30	095		33	571		37	161
	30	300		33	688		37	283
29	30	824	32	33	806	35	37	405
	30	438		33	924		37	527
	30	553		34	042		37	649
	30	667		34	161		37	771
	30	782		34	279		37	894
	30	897		34	397		38	017
	31	012		34	516		38	140
	31	127		34	635		38	263
	31	242		34	754		38	386
	31	357		34	873		38	509
30	31	473	33	34	992	36	38	633

M	G.	Par	M	G.	Par	M	G	Par
36	38	633	39	42	415	42	46	362
	38	757		42	544		45	496
	38	880		42	673		46	631
	39	004		42	802		46	766
	39	129		42	931		46	902
	39	253		43	061		47	037
	39	377		43	191		47	173
	39	502		43	320		47	309
	39	627		43	451		47	445
	39	752		43	581		47	581
37	39	877	40	43	711	43	47	718
	40	002		43	842		47	855
	40	128		43	973		47	992
	40	253		44	104		48	129
	40	379		44	235		48	267
	40	505		44	366		48	404
	40	631		44	498		48	542
	40	757		44	630		48	681
	40	884		44	762		48	819
	41	011		44	894		48	958
38	41	137	41	45	026	44	49	097
	41	264		45	159		49	236
	41	392		45	292		49	375
	41	519		45	425		49	515
	41	646		45	558		49	655
	41	774		45	691		49	795
	41	902		45	825		49	935
	42	030		45	959		50	076
	42	158		46	093		50	217
	42	287		46	227		50	358
39	42	415	42	46	362	45	50	499

M	G.	Par	M	G.	Par	M	G	Par
45	50	4 9	48	54	860	51	59	481
	50	641		55	010		59	540
	50	783		55	160		59	800
	50	925		55	310		59	960
	51	068		55	460		60	120
	51	210		55	611		60	280
	51	353		55	762		60	441
	51	496		55	913		60	602
	51	639		56	065		60	763
	51	783		56	217		60	925
46	51	927	49	56	369	52	61	088
	52	071		56	522		61	250
	52	215		56	675		61	413
	52	360		26	828		61	577
	52	505		56	981		61	740
	52	650		57	135		61	904
	52	795		57	289		62	069
	52	941		57	444		62	234
	53	087		57	598		62	399
	53	233		57	754		62	564
47	53	380	50	57	909	53	62	730
	53	526		58	065		62	897
	53	673		58	221		63	063
	53	821		58	377		63	231
	53	968		58	534		63	398
	54	116		58	691		63	566
	54	264		58	848		63	734
	54	413		59	006		63	903
	54	562		59	164		64	072
	54	711		59	322		64	242
48	54	860	51	59	481	54	64	412

M	G.	Par	M	G.	Par	M	G.	Par
54	64	412	57		711	60	75	456
	64	582		69	895		75	656
	64	753		70	080		75	857
	64	924		70	263		76	059
	65	096		70	449		76	261
	65	268		70	635		76	464
	65	440		70	821		76	667
	65	613		71	008		76	871
	65	786		71	195		77	076
	65	960		71	383		77	281
55	66	134	58	71	572	61	77	487
	66	308		71	761		77	694
	66	483		71	950		77	901
	66	659		72	140		78	109
	66	835		72	331		78	317
	67	011		72	522		78	526
	67	188		72	714		78	736
	67	365		72	906		78	947
	67	543		73	099		79	158
	67	721		73	292		79	370
56	67	900	59	73	486	62	79	583
	68	079		73	680		79	796
	68	258		73	875		80	010
	68	438		74	071		80	225
	68	618		74	267		80	441
	68	799		74	464		80	657
	68	981		74	661		80	874
	69	163		74	859		81	091
	69	345		75	057		81	310
	69	528		75	256		81	529
57	69	711	60	75	456	63	81	749

M	G.	*Par*	*M*	G.	*Par*	*M*	Gr.	*Pa*
63	81	749	66	88	735	69	96	575
	81	970		88	971		95	854
	82	191		89	219		97	135
	82	413		89	467		97	418
	82	635		89	716		97	701
	82	860		89	967		97	986
	83	084		90	218		98	272
	83	310		90	470		58	568
	83	536		90	723		98	849
	83	763		90	978		99	139
64	83	990	67	91	232	70	59	431
	84	219		91	489		99	724
	84	448		91	746		100	018
	84	678		92	005		100	314
	84	909		92	264		100	612
	85	141		92	525		100	910
	85	374		92	787		101	211
	85	607		93	050		101	513
	85	842		93	314		101	816
	86	077		63	579		102	121
65	86	318	68	93	846	71	102	427
	86	550		94	113		102	735
	86	788		94	382		103	044
	87	027		94	652		103	356
	87	267		94	923		103	668
	87	508		95	195		103	983
	87	749		95	468		104	229
	87	992		95	743		104	616
	88	235		96	019		104	936
	88	480		96	296		105	257
66	88	725	69	96	575	72	105	579

M	*Gr.*	*Par*	*M*	*Gr.*	*Par*	*M*	*Gr.*	*Par*
72	105	579	75	116	171	78	129	075
	105	904		116	559		129	558
	106	230		116	949		130	045
	106	558		117	342		130	536
	106	888		117	737		131	031
	107	220		118	135		131	530
	197	553		118	536		132	034
	107	888		118	639		132	542
	108	226		119	345		133	055
	108	565		119	755		133	572
73	108	906	76	120	166	79	134	094
	109	249		120	581		134	620
	109	594		121	000		135	151
	109	941		121	420		135	687
	110	290		121	843		136	228
	110	641		122	270		136	775
	110	994		122	700		137	326
	111	349		123	133		137	883
	111	707		123	570		138	445
	112	066		124	009		139	012
47	112	428	77	124	452	80	139	585
	112	792		124	898		140	164
	113	158		125	848		140	748
	113	526		125	801		141	339
	113	897		126	258		141	936
	114	270		126	718		142	538
	114	645		127	182		143	147
	115	023		127	649		143	763
	115	403		128	121		134	385
	115	786		128	596	81	145	014
75	106	171	78	129	075		145	650

M	Gr.	Par	M	Gr.	Par	M	Gr.	Pa
81	145	650	84	168	947	87	208	705
	146	292		169	912		210	649
	146	940		170	893		212	668
	147	600		171	891		214	745
	148	265		172	907		216	909
	148	937		173	941		219	158
	149	618		174	994		221	498
	150	307		176	067		223	938
	151	003		177	160		226	486
	151	709		178	275		229	153
82	152	423	85	179	411	88	231	950
	153	147		180	569		234	891
	153	878		181	752		237	991
	154	620		182	960		241	268
	155	372		184	194		244	744
	156	132		185	454		248	445
	156	903		986	743		252	402
	157	685		188	062		256	652
	158	478		189	411		261	243
	159	281		190	793		266	235
83	160	096	86	192	210	89	271	705
	160	922		193	661		277	753
	161	761		195	151		284	517
	162	612		196	680		292	191
	163	475		198	251		301	058
	164	352		199	867		311	563
	165	242		201	529		324	455
	166	146		203	240		341	196
	167	065		205	005		365	039
	167	999		206	825		408	011
84	168	947	87	208	705	90	Infinit	

Probl. 42. *The latitudes of two places together with their difference of longitude, being knowne, to finde the Rumb directing from the one to the other.*

As the Meridionall difference is to the difference of *longitude*; ſo is the Radius to the tangent of the *Rumb*.

Probl. 43. *By both latitudes and Rumb, to finde the diſtance upon the Rumb.*

As the Coſine of the Rumb, to the true difference of Latitudes; ſo is the Radius to the diſtance required.

Probl. 44. *By both latitudes and Rumb, to finde the difference of longitude.*

As the Radius is, to the Tangent of the Rumb; ſo is the Meridionall difference of the latitudes, to the difference of longitude required.

Probl. 45. *By both latitudes and diſtance, to finde the Rumb.*

As the diſtance is to the difference of Latitude; ſo is the Radius to the Coſine of the Rumb.

M	*Gr.*	*Par*	*M*	*Gr.*	*Par*	*M*	*Gr.*	*Pa*
81	145	650	84	168	947	87	208	705
	146	292		169	912		210	649
	146	940		170	893		212	668
	147	600		171	891		214	745
	148	265		172	907		216	909
	148	937		173	941		219	158
	149	618		174	994		221	498
	150	307		176	067		223	938
	151	003		177	160		226	486
	151	709		178	275		229	153
82	152	423	85	179	411	88	231	950
	153	147		180	569		234	891
	153	878		181	752		237	991
	154	620		182	960		241	268
	155	372		184	194		244	744
	156	132		185	454		248	445
	156	903		986	743		252	402
	157	685		188	062		256	652
	158	478		189	411		261	243
	159	281		190	793		266	235
83	160	096	86	192	210	89	271	705
	160	922		193	661		277	753
	161	761		195	151		284	517
	162	612		196	680		292	191
	163	475		198	251		301	058
	164	352		199	867		311	563
	165	242		201	529		324	455
	166	146		203	240		341	196
	167	065		205	005		365	039
	167	999		206	825		408	011
84	168	947	87	208	705	90	Infinit	

Probl. 42. *The latitudes of two places together with their difference of longitude, being knowne, to finde the Rumb directing from the one to the other.*

As the Meridionall difference is to the difference of *longitude*; ſo is the Radius to the tangent of the *Rumb*.

Probl. 43. *By both latitudes and Rumb, to finde the diſtance upon the Rumb.*

As the Coſine of the Rumb, to the true difference of Latitudes; ſo is the Radius to the diſtance required.

Probl. 44. *By both latitudes and Rumb, to finde the difference of longitude.*

As the Radius is, to the Tangent of the Rumb; ſo is the Meridionall difference of the latitudes, to the difference of longitude required.

Probl. 45. *By both latitudes and diſtance, to finde the Rumb.*

As the diſtance is to the difference of Latitude; ſo is the Radius to the Coſine of the Rumb.

Prob. 46. *By one latitude, diſtance, and Rumb, to finde the other latitude.*

As the Radius is, to the Coſine of the Rumb; ſo is the diſtance to the true difference of Latitude.

FINIS.

Faults in the Diſcourſe.

Pag.8 *line* 17 *for* 30102. *reade* 301029. *and line* 27. *for* ſome *reade* foure.

Pag.50. *line* 25. *for* 100000. *reade* 10000.

Pag. 58. *through the whole page, for* .9375, *reade* .09375.

Pag. 115. *line* 24. *for* at *reade* and.

And Pag. 126. *line* 5. *for* the ſide *reade* the ſame ſide.

Λογαριθμοτεχνία,

OR

The Conſtruction, and uſe *of the Logarithmeticall* TABLES;

By the help of which, Multiplication is performed by Addition, Diviſion by Subſtraction, the extraction of the Square root by bipartition, and of the Cube root by tripartition, &c. Finally, the Golden rule, and the reſolution of Triangles, as well right lined, as Sphericall by Addition and Subſtraction.

Firſt publiſhed in the French Tongue by *Edmund Wingate*, an Engliſh Gentleman:

AND

After tranſlated into Engliſh by the ſame Author.

The ſecond Edition, diligently corrected, and much enlarged by the author himſelfe.

Ἐὰν ἦς φιλομαθὴς, ἔσῃ πολυμαθής.

LONDON,
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M. DC. XXXV.

have the Logarithme of the fourth terme demanded.

Example. If a *Bullet*, whoſe *Diameter* is 4 inches, weigh 9 pound; another *Bullet*, whoſe *Diameter* is 8 Inches, will weigh 72 pound.

Diameter	4	0,602060
Diameter	8	0,903090
Difference		301030
Difference tripled		903090
Weight given	9	0,954243
Weight required	72	1,857333

But if the firſt terme be greater than the ſecond, ſubſtract the Difference tripled from the Logarithme of the other terme.

Example,

Diameter	8	0,903090
Diameter	4	0,602060
Difference		301030
Difference tripled		902090
Weight given	72	1,857333
Weight required	9	0,954243

PROBL.

PROBL. VII.

Any number being given to finde the ſquare root thereof.

DIvide the Logarithme of the number given by 2; this done, you have the Logarithme of the Root required.

Example,

The number given	12572.	4,099404
The Root required	112.12	2,049702

PROBL. VIII.

To finde the Cube root of any number given.

DIvide the Logarithme of the number given by 3; this done, you have the Logarithme of the Root required.

Example,

The number given	79304.	4,899295
The Root required	42.953	1,633098

PROBL. IX.

Betwixt two numbers given to finde a meane Proportionall.

ADde the Logarithmes of the two numbers given together, then dividing their ſumme by 2, *you have the Logarithme of the Meane Proportionall required.*

Example,

The numbers given	8	0,903090
	32	1,505150
The ſumme of their Log.		2,408240
The halfe ſumme		1,204120

which is the Logarithme of 16, the *Meane proportionall* demanded.

PROBL. X.

Betwixt two numbers given to finde two meane Proportionals.

Having ſound the difference betwixt the Logarithmes of the numbers given, and added a third part thereof to the Logarithme of the leaſt number propounded, you have the Logarithme of the leſſer meane Proportionall you look for; to which if you againe adde the ſame third part, you ſhall have the Logarithme of the other meane Proportionall required.

Example,

The numbers given	8	0,903090
	64	1,806180
The difference		903090
The third part		301030
The leſſer m. prop.	16	1,204120
The greater m.prop.	32	1,505150

PROBL.

PROBL. XI.

A ſumme of money being forborne for a certaine time, to finde how much it will be augmented at the expiration of the ſame time, accounting intereſt upon intereſt according to a certaine rate propounded.

SVbſtract the Logarithme of 100 *from the Logarithme of* 100 *and the rate added together; this done, if you multiply their difference with the time propounded, and then adde that Product unto the Logarithme of the Principall, that ſumme is the Logarithme of the Principall and Intereſt required.*

Example. I deſire to know how much 2735, *l.* being forborne 11 yeares, will be increaſed at the expiration of thoſe yeares, accounting Intereſt upon Intereſt after the rate of 8 *l. per centum:* Here if I ſubſtract 2,000000 (the Logarithme of 100) out of 2,033424 (the Logarithme of 108) the remainder is 33424, which being multiplied by a 11 yeares, produceth 367664; wherefore if I now adde the ſame 367664 to 3,436957 the Logarithme of 2735 (the *Principall*, or ſumme of mony forborn) their *Aggregate* is 3,804621, which is the Logarithme of 6377.1, the *Principall increaſed*: So that I conclude, 2735, *l.* being forborne 11 years will augment to 6377, *l.* 2, *s.* (for $\frac{1}{10}$ of a pound is 2 *ſhillings*) accounting Intereſt upon Intereſt according to the rate of 8, *l. per centum*.

PROBL. XII.

A ſumme being due at a time to come, to finde what it is worth in readie money.

HEre the worke is the ſame with that of the former Probleme, only at laſt inſtead of adding, you are to ſubſtract the Product out of the Logarithme of the Principall; for that performed, the remainder is the Logarithme of the ſumme you looke for.

Example. What is 2735, *l.* due to be paid 11 yeares hence, worth now in ready money? Here having found the *Product* 367664, as before, I deduct it out of 3,436957, and finde the remainder to be 3,069293, which is the Logarithme of 1173, *l.* the ſumme demanded.

PROBL. XIII.

The Principall, and the Aggregate of the Principall and Intereſt together being given, to finde the rate of the Intereſt.

SUbſtract the Logarithme of the Principall out of the Logarithme of the Aggregate; this done, if you firſt divide the Difference by the time, and then adde that Quotient to the Logarithme of 100, *that ſumme is the Logarithme of* 100, l. *and the rate of the Intereſt added together.*

A delivers to *B* 230, *l.* with *caution* to receive in *lieu* thereof at the expiration of 21 yeares 850, *l.* and in the *Interim* to forbeare the Intereſt; the *Queſtion* is, after what rate of Intereſt *B* is to hold this money: Here firſt 2,361728 (the Logarithme of 230) being deducted out of 2,929419, (the Logar. of 850) their *Difference* is 567691, which if I divide by 21, the *Quotient* is 27033; This *Quotient* if I adde to 2,000000 (the Logarithme of 100) their ſumme or *Aggregate* is 2,027033, which is the Logarithme of 106.42 (*viz.* 106, *l.* and $\frac{42}{100}$ of a pound, which Fraction being reduced (by the *Table* of *Reduction* produced in the ſeavenih *Probleme* of the laſt Chapter) is 8, *s.* 4. *d.* 3, *farth.* I conclude therefore that *B* retaines the 230. l. for the time aforeſaide at the *rate* of 6, *l.* 8, *s.* 4, *d,* 3 *fa. per centum*, which is the reſolution of the *Probleme* propounded.

PROBL. XIV.

A yearely rent or annuity being forborne a certaine number of yeares, to finde what the averages thereof will amount unto, according to any rate propounded.

First diſcover the Principall, that anſwers to the rent or annuity in queſtion; then finde unto what ſumme that Principall will bee augmented (according to the given rate) at the end of the terme propounded; this done, if you ſubtract the ſame Principall out of that ſumme, the remainder is the ſumme of the Averages you looke for.

Example, A rent or Annuity of 12, l. *per annum* being forborne 16 yeares, what will the *Averages* thereof amount unto, they being conceived to increaſe (as they grow due) after the *rate* of 8, l. *per centum*? Here firſt to find the *Principall* that anſwers to 12, l. ſay *thus*,

If 8, l. hath 100, l. for his *Principall*, what ought 12, l. to have for his correſpondent *Principall?*

The anſwere will bee (by the third *Probleme* of this Chapter) 150, l. for the reſpective *Principall* of 12, l. Having thus diſcovered the *Principall* of 12, l. *viz.* 150, l. I find (by the 11. *Probleme* afore going) that the ſame 150, l. being forborne 16 years, will amount (after the rate of 8, l. *per centum*) to 513. 89, that is, 513, *l.* 17, *s.* 9, *d.* now therefore if I deduct 150 (the correſpondent *Principall* of the Annuity given) out of the ſame 513, *l.* 17, *s.* 9, *d.* The remainder, *viz.* 363, *l.* 17, *s.* 9, *d.* is the ſumme of the *Averages* required.

PROBL. XV.

A yearely rent or Annuity, being propounded, to find what it is worth in ready money.

First finde what the Averages thereof amount unto at the end of the terme propounded, And then what thoſe averages are worth in ready money

money, which ſhall likewiſe be the required price or value of the rent or annuity propounded.

Example, what may a man, which is deſirous to lay out his money after the *rate* of 8, *l. per centum*, afford to give for a leaſe of **12.** *l.* ***per annum***, that hath yet **16** yeares in being? I finde (by the laſt Probleme) that the Arerages of **12.** ***l. per annum***, being forborne **16** years, amount then to **363,** ***l.*** **17,** ***s.*** **9,** ***d.*** or **393.89**, and I find likewiſe (by the twelveth *Probleme* aforegoing) that the ſame **363,** ***l.*** **17,** ***s.*** **9,** ***d.*** is worth in preſent mony **106.21.** *l.* or which is all one) **106.** ***l.*** **4,** ***s.*** **2,** ***d.*** I conclude therefore, that the *value* of the leaſe propounded (at the rate of **8,** *l.* ***per centum***) is **106,** *l.* **4,** *s.* **2.** *d.*

Here when the terme of the Annuity beginnes not preſently, but after certaine yeares to come, Find what the arerages forborne for all the time are worth in ready money.

So in the *Example* laſt premiſed, if the annuity, of 16, *yeares* were not to beginne till after the expiration of 5 *yeares*, In this caſe you are to inquire what the *arerages*, (*viz.* 363, l. 17, s. 9, d.) being forborne **21.** *yeares*, are worth in ready mony, which you ſhall likewiſe find (by the twelveth *Probleme* before cited) to bee 72. 288, which being reduced is 72, *l.* 5. *s.* 9, *d.* the *value* of the leaſe required.

PROBL. XVI.

A ſumme of money being propounded, to finde what Annuity, (to continue any number of yeares, and according to any rate given) that ſumme will buy.

TAke any annuity at pleaſure; then finde the value of that annuity in ready money: This done, the proportion will be, as followeth,

As the value found is to the annuity taken; So is the ſumme given to the annuity required.

Example, what *annuity* (to continue 16 yeares) will 1205, l. deſerve, ſo that the purchaſer may gaine after the rate of 8. l. *per centum?* Here firſt I take 12. l. *per annum* to continue 16 yeares, and find the *value* thereof in ready mony (by the 15 *Probleme* of this Chapter) to be 106. 21, or 106, *l.* 4, *s.* 2, *d.* I ſay therefore,

If 106.21, *l.* give 12, *l. per annum.*

What will 1205, *l.* yeild? *Facit* 171. 39, *per annum* which being reduced is 171, *l.* 7, *s.* 9, *d.*

9,*d.* I conclude therefore that 171,*l.*7,*s.* 9,*d.* is the *Annuity* (to endure 16 yeares) which 1205,l. doth deſerve, after the rate of 8. *per centum.*

If any deſire to have theſe *Problemes* of *Intereſt* farther exemplified, let them turne to the two laſt Chapters of the *Appendix* of my *Arithmetique.*

PROBL. XVII.

Any number of Souldiers being given, to order them into a ſquare Battaile of men.

TAke halfe the Logarithme of the number given; this done, that halfe is the Logarithme of the number of men, that ought to be placed in Ranke *and File to make a ſquare Battaile of men.*

Example, 573. Souldiers are propounded to be thus ordered; Here the Logarithme of 573 is 2,758155, whoſe *halfe* is 1,379077 *viz.* the Logarithme of 23.937; I ſay therefore, that 23 men ought to bee placed in *Ranke*, and as many likewiſe in *File* to make a *ſquare Battaile* of the 573 Souldiers propounded: And here *obſerve*, that the *Decimall* ·937 is altogether neglected, becauſe *Fractions* are not to be conſidered in queſtions, that concerne *Military* orders.

PROBL. XVIII.

Any number of Souldiers being propounded, to order them into a double battaile of men, viz. *which may have twice ſo many in Ranke, as in File.*

TAke halfe the Logarithme of halfe the number given; this done, that halfe Logarithme is the Logarithme of the number of men to be placed in File, and the ſame number doubled will ſhew you how many ought to be placed in Ranke.

Example, 1342 Souldiers being propounded to bee put into that *order*, I demaund how many ought to bee placed in *Ranke*, and how many in *File*: Here *halfe* 1342 is 671, whoſe Logarithme is 2,826723, which being *halfed* is 1,413361, whoſe *correſpondent* number is 26, &c. I conclude therefore, that to order 1342 Souldiers into a *double Battaile*, you are to place 26 men in *File*, and 52 in *Ranke*.

PROBL.

PROBL. XIX.

Any number of Souldiers being given, to order them into a quadruple Battaile, viz. *ſuch as may have foure times ſo many in Ranke, as in File.*

TAke halfe the Logarithme of a fourth part of the number given; this done, that halfe Logarithme is the Logarithme of the number of men to be placed in File, and the ſame number being multiplied by 4 will ſhew you how many men ought to be placed in Ranke.

Example, 2048 Souldiers being offered to be put into this *order*; Here the fourth part of 2048 is 512, whoſe Logarithme is 2,709270 which being halfed is 1,354635, whoſe proper number is 22, &c. I ſay therefore that, to order 2048 Souldiers into a quadruple Battaile, 22 men ought to be placed in *File*, and 88 in *Ranke*. And here note that to diſtinguiſh *this* from other *orders*, the *French* call it, *Bataillon de grand front*, from whence likewiſe we uſually terme it a *Bataile of the great Front*.

PROBL. XX.

Any number of Souldiers being propounded, to order them in Ranke and File according to the Reaſon of any two numbers given.

ADde the Logarithme of the given number of Souldiers, to the Logarithme of the proportionall number given for the Ranke; this done, if out of their Aggregate you deduct the Logarithme of the proportionall number given for the File, halſe the remainder is the Logarithme of the number of men to be placed in Ranke, which Logarithme if you ſubtract out of the Logarithme of the whole number of Souldiers, the remainder is the Logarithme of the number to be placed in File.

Example, 2500 Souldiers are ſo to be ordered, that the number of men placed in *File* may beare ſuch *Proportion* to the number of men placed in *Ranke*, as 5 beare to 12; In this demaund the *Proportion* will be this, *as 5 to 12, ſo 2500 to another number, whoſe ſquare roote is the number of men to bee placed in Ranke*: Wherefore if you adde 1, 079181 the

Logarithme of 12, to 3,397940 the Logarithme of 2500, their *Aggregate* is 4,477121, out of which if you ſubtract 0,698970 the Logarithme of 5, the remainder is 3,778151, whoſe *halfe* being 1,889075 is the Logarithme of 77, &c. the number of men to bee placed in *Ranke*; and now if out of 3,397940 the Logarithme of 2500 you deduct 1,889075 the Logarithme of the number of men in *Ranke*, the remainder (*viz.* 1,508865) is the Logarithme of 32, &c. the number of men to be placed in *File*.

PROBL. XXI.

Any number of Souldiers being given, together with their diſtances in Ranke and File, to order the ſame Souldiers into a ſquare Battaile of Ground.

ADde the Logarithme of the diſtance in Ranke to the Logarithme of the number of Souldiers; this done, if out of their ſumme you ſubtract the Logarithme of the diſtance in File, halfe of that which remaines is the Logarithme of the number of men to be placed in File, which

Logarithme being deducted out of the Logarithme of the whole number of Souldiers, the remainder is the Logarithme of the number of men to be placed in Ranke.

Example, 2500 men are propounded to be ſo martialled into a *ſquare Battaile* of *ground*, that, their diſtance in *File* being 7 foot and their diſtance in *Ranke* 3 foot, the ground whereupon they ſtand may bee juſt *ſquare:* Here the *Proportion* will be this, *As 7 to 3, ſo 2500 to another number, whoſe ſquare roote is the number of men to be placed in File:* Wherefore if you adde 0,477121 the Logarithme of 3 (the diſtance in *Ranke*) to 3,397940 the Logarithm of 2500, their ſumme is 3,875061, out of which if you ſubtract 0,845098 the Logarithme of 7, the diſtance in *File*, the remainder is 3,029963, whoſe *halfe* being 1,514981 is the Logarithme of 32, &c. the number of men to be placed in *File*; and now if out of 3,397940 the Logarithme of 2500 you ſubtract 1,514981 the Logarithme of the number of men in *File*, that which remaines, *viz.* 1,882959, is the Logarithme of 76, &c. the number of men to bee placed in *Ranke*. And *note*, that this *order* is termed a *ſquare Battaile*, as well as that mentioned in the ſeaventeenth *Probleme* aforegoing, yet is there much difference betwixt them, For in a *ſquare Battaile of men* the number of men in *Ranke* is alwayes equall to the number of men in *File*, though the *ground*, whereon they ſtand, bee not alwayes a juſt *ſquare*; contrariwiſe, in a *ſquare Battaile of ground* it is neceſſary, that the place, on which they ſtand, ſhould bee alwayes a juſt *ſquare*, though the number of men in *Ranke* be not equall to the number of men in *File*: And therefore the *French* (as well as we) call that order *Batailon quarrê d'hommes*, a ſquare Battaile of men, and this, *Bataillon quarré de Terrain*, a ſquare Battaille of Ground.

PROBL. XXII.

Any number of Souldiers being propounded together with the diſtances in Ranke and File, to order them in Ranke and File according to the reaſon of any two numbers given.

Having added the Logarithmes of the diſtance in File and of the proportionall number given for the Ranke to the Logarithme of the number of Souldiers propounded, ſubtract out of that Aggregate the ſumme of the Logarithmes of the diſtance

ſtance in Ranke and of the proportionall number given for the File; this done, halfe that which remaines is the Logarithme of men to bee placed in Ranke, which Logarithme being ſubtracted out of the Logarithme of whole number of Souldiers, the remainder is the Logarithme of the number of men to be placed in File.

Example, 12750 are ſo to bee ordered, that the diſtance betwixt man and man may be 7 foot upon the *File* and 3 foot upon the *Ranke*, and that the length of ground for the *Ranke* may beare the ſame *proportion* to the length of ground for the *file*, that 12 beares to 5; Here the Logarithme of 7 (the diſtance in *file*) is 0,845098; the Logarithme of 12 (the *proportionall* number given for the *ranke*) is, 1,079181; and the Logarithme of 12750 (the number of Souldiers propounded) is 4,105510; the *aggregate* of theſe three Logarithmes is 6,029789; Againe the Logarithme of 3 (the diſtance in *ranke*) is 0,477121, and the Logarithme of 5 (the *proportionall* number given for the *ranke*) is 0,698970; theſe two laſt Logarithmes being added together, their *ſumme* is 1,176091, which being deducted out of the *aggregate* of the three former Logarithmes, the remainder is 4,853698, whoſe *halfe* being 2,426849 is the Logarithm of 267, &c. the number of men to be placed in *ranke*; And now if I ſubtract the ſame Logarithme 2,426849 out of 4,105510 the Logarithme of 12750, the number of Souldiers propounded, the remainder, (*viz.* 1,678661) is the Logarithme of 47, &c. the number of men to be placed in *File*.

CHAP. VI.

The uſe of the Logarithmeticall Tables in the reſolution of right line Triangles.

THus much concerning the uſe of the Logarithmeticall Tables in Arithmetike, now followes their uſe in Geometry, which is chiefly diſcovered in the reſolution of Triangles: And now according to the diviſion of Triangles into right lined, and Sphericall, wee will firſt ſhew their uſe in the reſolution of right line Triangles.